Tense Probabilism Properly Conceived

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1 Introduction

"Tense Probabilism" (henceforth TP) is Barker's (1998) term for the doctrine that (i) the probabilities of conditionals are conditional probabilities and (ii) differences in probability between different "versions" of conditionals are due to differences in temporal reference. One of its tenets can be paraphrased as follows:

Thesis 1 (Tense Probabilism)
The value of a counterfactual at a given time is that of its indicative predictive counterpart at an earlier time.[1]

Thus assuming that Oswald killed Kennedy and that he acted alone, (1b) is unlikely now because (1a) *was* unlikely on the morning of November 22, 1963.

(1) a. If Oswald does not kill Kennedy, someone else will.
  b. If Oswald had not killed Kennedy, someone else would have.
  c. If Oswald did not kill Kennedy, someone else did.

Many authors have made this observation and incorporated it in their semantic accounts in various ways (e.g., Adams, 1975; Skyrms, 1981; Tedeschi, 1981; Ellis, 1984; Dudman, 1984, 1994; Edgington, 1995; Dahl, 1997; Dancygier, 1998).

Notice that the value of (1c) is not as clearly related to either one of the others. Here I will focus on the relationship between (1a) and (1b).

The problem

The above formulation of Thesis 1 leaves room for interpretation. The question of interest here is which "values" of sentences like (1a,b) are claimed to be correlated, and how.

Barker (1998) reads "value" as "probability" and shows that under this reading Thesis 1 is at odds with both naïve intuitions about certain examples and the commitment to non-determinism implicit in TP.

That TP implicitly assumes non-determinism is shown as follows: Despite the fact that Oswald killed Kennedy, (1b) may have high probability. Given Thesis 1 (under Barker's reading), this implies that (1a) had high probability at an earlier time, which by TP must be the conditional probability (at the time) that someone else will kill Kennedy, given that Oswald doesn't. But if Oswald's killing Kennedy had been brought about deterministically, the antecedent of (1a) would never have had non-zero probability, hence the required conditional probability would not be defined and Thesis 1 would be refuted.

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[1] This statement is stronger than Barker's, which says that if the value of counterfactual is high now, then that of the indicative was high at an earlier time.

Barker argues that its alignment with non-determinism notwithstanding, TP makes the wrong predictions in situations involving genuine non-determinism. The argument involves sentences like (2a,b). Suppose the coin is fair and comes up heads.

(2) a. If I bet on tails, I will lose. [before the toss]
b. If I had bet on tails, I would have lost. [after the toss]

After the toss, since the coin came up heads, (2b) is highly probable. In contrast, before the toss, the probability that the coin would come up heads was .5, hence (2a) was not highly probable, barring the assumption (which is incompatible with TP) that the outcome was already determined.

Intuitively, the problem is that while the posterior fact that the coin came up heads is taken into account in interpreting (2b), it seems irrelevant to (2a); nor does there appear to be a coherent way within TP to bring it to bear on (2a). Barker concludes that TP has no promise of providing a unified account of counterfactuals and predictive indicatives. I am going to show that this conclusion is not inevitable.

2 Values and probabilities

Barker's is not the only reading of Thesis 1. I propose that a distinction be drawn between the *values* conditionals take at individual worlds on the one hand, and their *probabilities*, on the other. This distinction provides for an improved statement of Thesis 1 according to which conditionals like (2a,b) are *equivalent* though not *equiprobable*.

To prepare the ground for the argument, I will first pin down the relationship between truth values and probability. The basic framework is due to Jeffrey (1991); Stalnaker and Jeffrey (1994) and ultimately to De Finetti.

Let $\langle W, Pr\rangle$ be a set of worlds with a probability distribution $Pr : \wp(W) \mapsto [0,1]$, and let $X : W \mapsto \mathbb{R}$ be a *random variable* taking real numbers at the worlds in W. The *expectation* of X is as in (3).[2]

(3) $$E[X] = \sum_{x \in range(X)} x \cdot Pr(\{w \in W | X(w) = x\})$$

The denotations of atomic or truth-functional sentences, too, are random variables taking values in $\{0,1\}$ point-wise at worlds. The expectation of such a sentence is the probability that it is true. This connection is used to define a probability distribution P (distinct from Pr) over the sentences of the language. Let $V : \mathcal{L} \mapsto (W \mapsto \{0,1\})$ be a truth assignment to the language $\mathcal{L}$ of propositional logic. Then for each sentence $\varphi \in \mathcal{L}$, the probability $P(\varphi)$ is defined as the expectation of $V(\varphi)$; since the range of $V(\varphi)$ is $\{0,1\}$, this comes down to (4).

(4) $$P(\varphi) = \sum_{x \in range(V(\varphi))} x \cdot Pr(\{V(\varphi) = x\}) = Pr(\{V(\varphi) = 1\})$$

[2] Henceforth I abbreviate '$\{w \in W | X(w) = x\}$' as '$\{X = x\}$.' In the continuous case, the summation is replaced by an integral.

Extending the language to include the "natural" conditional '$\rightarrow$' (to be distinguished from the material conditional '$\supset$'), TP requires that the expectation of $V(A \rightarrow C)$ equal the conditional probability $P(C|A) = P(AC)/P(A)$ whenever the latter is defined. This has proven elusive. A long line of research, starting with the *triviality results* of Lewis (1976, 1986b) and summarized in Hájek and Hall (1994), has compiled overwhelming evidence that if the values of $V(A \rightarrow C)$ are to add up to the conditional probability, they cannot be truth values—assuming that the truth values of a sentence are, at each world, (i) either 0 or 1 and (ii) constant.

Rather than retracing these arguments here, I am going to build on an alternative proposal due to Jeffrey (1991) (cf. also van Fraassen, 1976; Stalnaker and Jeffrey, 1994): For (non-counterfactual) conditionals, the assignment is

(5) $$V(A \rightarrow C)(w) = \begin{cases} V(C)(w) \text{ if } V(A)(w) = 1 \\ E[V(C)|V(A) = 1] \text{ if } V(A)(w) = 0 \end{cases}$$

An intuitive rationale for (5) may be gleaned from Stalnaker's (1968) theory of counterfactuals. If A is false at w, the value of the conditional depends on the "nearest" A-world, uniquely identified in Stalnaker's models by a selection function. (5) does not require that a single nearest A-world be found; instead, the selection function picks an A-world *at random* and assigns the conditional its value according to the upper line in (5). The value of the conditional at w is the expectation of this random trial.

Notice that according to (5), at worlds where A is false, the values of the conditional may be intermediate between 0 and 1, and furthermore, those values depend on Pr (via the expectation), hence are not constant. If Pr is interpreted as *subjective* probabilities representing the beliefs of an agent, mixing these intermediate values with (objective) truth values calls for some explanation (cf. Edgington, 1995). I will not discuss this issue here since does not arise with predictive conditionals. Here Pr is interpreted as objective chance, fully determined by the world and time of evaluation (cf. Lewis, 1980).[3]

Notice further that according to (5), the values of the conditional are uniformly distributed over all non-antecedent worlds, independently of any "third facts." Figuratively speaking, the set of "visible" A-worlds over which the expectation is taken is the same for all $\overline{A}$-worlds. This is the root of the problem pointed out by Barker.[4]

3 Counterfactuals

In the literature on counterfactuals in time, the lesson from examples like (2) has long been recognized to be that "Past Predominance" (Thomason and Gupta, 1981) must be balanced against "Overall Similarity" (Lewis, 1979). (2) has

[3]Formally, this can be made explicit by enriching the model with a temporal dimension to which the value assignment is made sensitive. A "$T \times W$-frame" as defined by Thomason (1984) is a suitable structure, but in the interest of simplicity I am not going to spell out the details here (see Kaufmann, 2002).

[4]There are a number of other reasons to look for improvements of (5), some of which (Lance, 1991; Edgington, 1991) involve compounds of conditionals and fall outside the scope of this paper. The solution proposed here addresses those problems as well, as I will show on some other occasion.

been discussed in this connection by Slote (1978); Bennett (1984); Mårtensson (1999), among others.

The underlying question is whether actual facts at times later than that of the (hypothesized) antecedent should be "carried over" in examining alternative worlds. (2) suggests that they should: Had I bet on tails, the coin would still have come up heads and I would have lost. (1) suggests that they should not: Had Oswald not killed Kennedy, Kennedy would not have been killed.

The consensus is that the relevant distinction between examples like (2) and those like (1) concerns the *causal dependencies* of the respective scenarios: The outcome of the coin toss is causally independent of my not betting (notice that judgments about (2) change if, e.g., a different fair coin is used when I bet on tails), whereas Kennedy's fate causally depends on Oswald's behavior. Barker's objection, too, is properly addressed by augmenting the model with causal information.

4 Causality

Causal relations are taken here to hold between event *tokens* such that the cause determines the probability of the effect (Hausman, 1998; Pearl, 2000). At each world, the presence or absence of cause and effect are indicated by random variables (not necessarily denoted by sentences of the language). Thus 'X *causally affects* Y' means 'the *value* of X determines the *expectation* of Y.'[5]

Given the set W of worlds, a set Φ of functions $X : W \mapsto \{0, 1\}$ is singled out as the collection of *causally relevant* variables. The '*causally affects*'-relation is encoded as a strict partial order '$\prec$' on Φ. All *descendants* of a variable X in $\langle \Phi, \prec \rangle$ (i.e., all Y such that $X \prec Y$) are causally affected by X.

Φ induces a partition on the set of worlds, each of whose cells comprises those worlds which agree on the values of all variables in Φ. Crucially, the assignment of values to conditionals, counterfactual or not, starts "locally" within those cells. Let $\mathcal{C}$ be such a cell and w a world in $\mathcal{C}$. Suppose $V(A)$ is false at w. Then for a conditional $A \to C$, if the conditional expectation of C given A is defined within $\mathcal{C}$, it is the value of $V(A \to C)(w)$. This is similar to (5) above, but now the conditional expectation is taken only over those antecedent-worlds which agree with w on the causally relevant variables.

If, on the other hand, the conditional expectation is not defined within $\mathcal{C}$ (i.e., if A has zero probability given $\mathcal{C}$), a larger cell $\mathcal{C}'$ is made available by "undoing" the falsehood of $V(A)$.[6] This is where the causal order on variables matters: The values of the variables that are causally affected by $V(A)$ (its descendants in the causal order) are "given up" as well, whereas the other variables retain the values they have at w. Thus in (2), where no causal influence of the bet upon the outcome is assumed, from a world at which I do not bet on tails and the coin comes up tails, only those "tail-betting" worlds are visible in which the coin comes up tails as well. Similarly for heads.

This is visualized in Figure 1, where the coloring symbolizes the values assigned to (2a) before the coin toss. The values according to 5 (on the left) are

[5] If Y causally depends on additional variables besides X, its expectation is determined by a joint setting of *all* those causal factors, including X.

[6] This resembles the "fattening" of Skyrms (1984, 1994), but there are differences when there are multiple mutually exclusive ways of making A true; I will not discuss this case here.

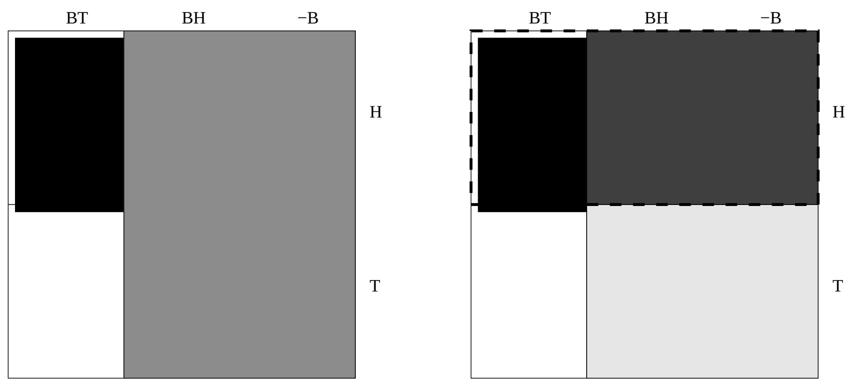

Figure 1: Distribution of values of (2); Black = 1; White = 0; Grey = intermediate, without (left) and with (right) causal information

distributed uniformly over all worlds at which I do not bet on tails, regardless of whether the coin comes up heads or tails. On the right, the distribution of values with reference to causality is given: At different non-antecedent worlds the conditional takes different values depending on the result of the toss.

To augment the interpretation function accordingly, the change to the clause for non-antecedent worlds in (5) consists in adding restrictions to the conditional expectation for worlds at which the antecedent is false: Besides '$V(A) = 1$', '$X = x$' is added for each *non-descendant* X of $V(A)$ in $\langle \Phi, \prec \rangle$ such that $X(w) = x$.

(6) $$V(A \to C)(w) = \begin{cases} V(C)(w) \text{ if } V(A)(w) = 1 \\ E[V(C)|V(A) = 1, X_i = x_i] \\ \quad \text{for all } X_i \in \Phi \text{ s.t. } V(A) \not\preceq X_i \text{ and } X_i(w) = x_i, \\ \quad \text{otherwise} \end{cases}$$

5 Conclusion

I have proposed a value assignment which makes reference to posterior facts (at times later than that of the antecedent) in the case of predictive conditionals. Is this legitimate? To see in what sense it is, we need to keep apart the notions of *truth* and *settledness*, familiar since Prior (1967) and Thomason (1970).

At the time the prediction in (2a) is made, there is no way of knowing which way the coin will land, thus both outcomes are open possibilities. Worlds of both types are indistinguishable with respect to past and present facts, so the uncertainty is objective and irreducible. It is not *settled* that the coin will come up heads; yet at each individual world it is already either *true* or *false*.

A similar statement can be made about conditionals: Although it is not *settled* what the actual value of the conditional is, at each individual world that value is already determined by the facts.

Thus as a predictive conditional "turns into" a counterfactual, it retains its values at individual worlds while its probability (i.e., the expectation of its values) may change simply due to the elimination of historical alternatives.

Before the toss, the value of (2a) is high at those non-antecedent worlds at which the coin comes up heads. Once the outcome (heads) is settled, only those worlds remain at which the value was high *already*. Hence the above restatement of Thesis 1 to the effect that the predictive indicative and its counterfactual counterpart are *equivalent* but not *equiprobable*.

Finally, an improved statement of the relationship between the probabilities of the two sentences becomes available as well: The probability of the indicative is the expectation of the probabilities that the corresponding counterfactual will have in the various cells of the partition induced by the causally related factors. Thus the probability of (2a) is the weighted sum of the probabilities of (2b), where the weights are the prior probabilities that (2b) will take those values.

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