NORTHWESTERN
UNIVERSITY

The Role of Hearers' Beliefs in the Interpretation of Logical Connectives

James German, Eyal Sagi, Brady Clark,

Stefan Kaufmann, and Min-Joo Kim

Overview

- Logical Connectives and Gricean Implicature
- Game Theory and the Gricean Model
- Experimental Game-Theoretic Approach
- Case Study
- Other Approaches
- Experiment

Logical Connectives and Implicature

Gricean/Neo-Gricean View

(Horn 1972, Grice 1975, Gazdar 1979)

- Connectives have a single sense (e.g., $\vee$)
- For a proposition Φ there is a proposition Ψ related to Φ in the following ways:
  - Ψ entails and/or is more informative than Φ
  - Φ and Ψ can be expressed by forms of approximately equal length

For example: (Φ) *The strawberry is next to an apple **or** a lemon.*

(Ψ) *The strawberry is next to an apple **and** a lemon.*

Logical Connectives and Implicature

Maxim of Manner *(Grice 1975)*

Be perspicuous: (i) avoid obscurity, (ii) avoid ambiguity, (iii) be brief, (iv) be orderly.

Maxim of Quantity *(Grice 1975)*

(i) Make your contribution as informative as is required for the current purposes of the exchange.

(ii) Do not make your contribution more informative than is required.

Logical Connectives and Implicature

- Speaker has said that Φ.
- Since Φ, Ψ are of roughly equal length/effort, 3rd Maxim of Manner (brevity) does not apply.
- If the speaker had known Ψ, he/she would have violated the 1st Maxim of Quantity.
- Therefore, the hearer may infer that ~Ψ.
- Sauerland (2004)

Logical Connectives and Implicature

Assumption: Two interpretations are available

*The strawberry is next to an apple **or** a lemon.*

(i) $(p \vee q)$

(ii) $(p \vee q) \wedge \sim(p \wedge q)$

*The strawberry is **not** next to an apple **and** a lemon.*

(i) $\sim(p \wedge q)$

(ii) $\sim(p \wedge q) \wedge \sim(p \vee q)$

*The strawberry is next to an apple **if** it is next to a lemon.*

(i) $(q \rightarrow p)$

(ii) $(q \rightarrow p)$, *indirectness condition* (Grice 1975): speaker has non-truth-functional grounds for the assertion (i.e., not true just because p is true or q is false)

Game Theory and Conversational Implicature

Cooperative Principle: Make your contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1975)

Game Theory and Conversational Implicature

(Neo)Gricean Assumptions:

- Expressions the speaker could have used but didn't
- Cooperativity: communicative goals are shared
- Act to maximize informativity, minimize effort
- Above are mutual beliefs

GT Principles:

- Choice among various possible actions
- Coordination: payoff structure is shared
- Payoffs, costs, rationality
- Above are mutual beliefs

Gricean Model as a Special Case

- Assumes cooperativity
  - Payoff structures not always shared
    (e.g., speaker may want to mislead, conceal, etc.)
- Cost is equated with length
  - Structural complexity, accessibility
- Assumes rationality
  - Evidence from economics, psychology
- Mutual beliefs
  - Mutuality varies with situation
  - Uncertainty

Experimental Game Theoretic Approach

- Treat the standard neo-Gricean assumptions as variables
- Use game-theoretic paradigm to systematically manipulate those variables
- Establish dependencies between variables

Experimental Game Theoretic Approach

Other GT approaches to Gricean/neo-Gricean pragmatics:

- Parikh (1992, 2001)
- van Rooij (2004)
- Stalnaker (2005)
- Jäger (2006)

Game Theory and Conversational Implicature

(Neo)Gricean Assumptions:

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GT Principles:

- Choice among various possible actions
- Coordination: payoff structure is shared
- Payoffs, costs
- Above are mutual beliefs

Games in Extensive Form

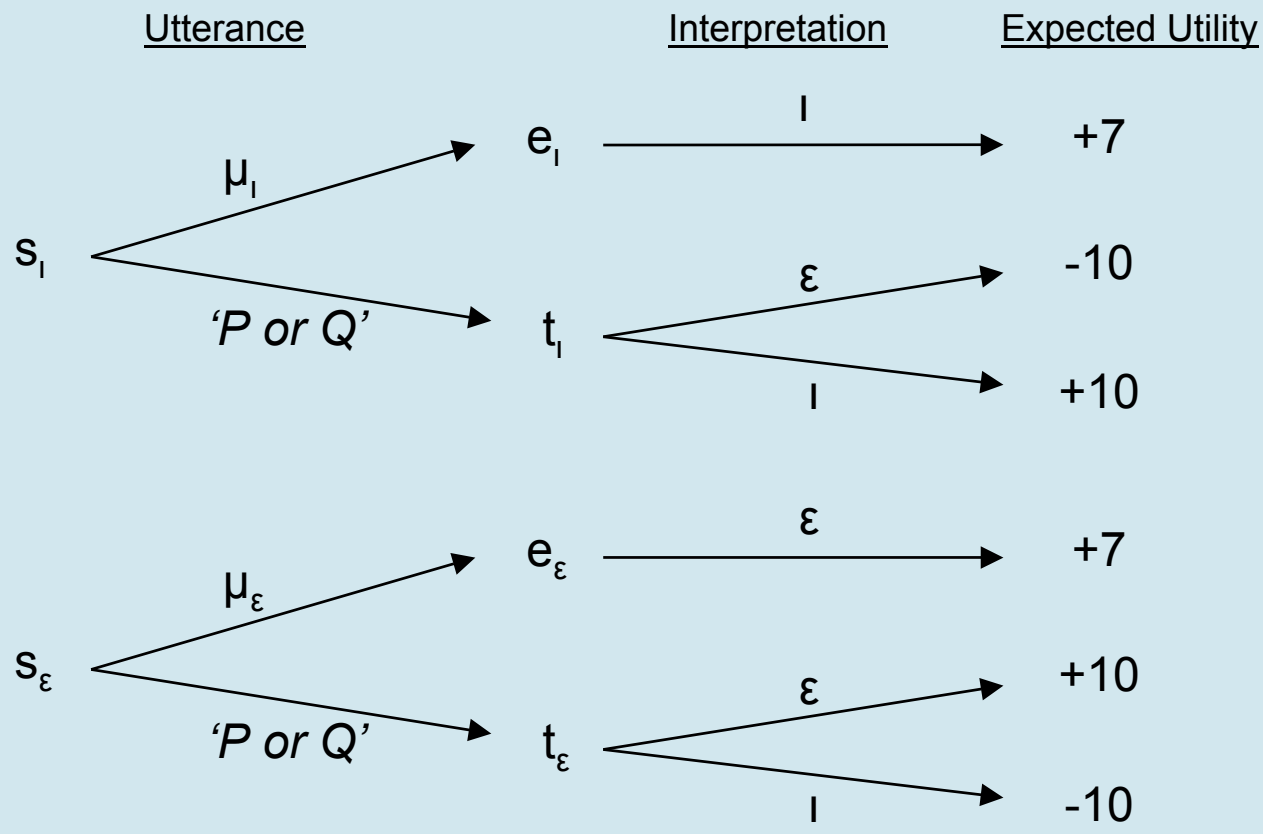
Interlocutors share three types of information (Parik 2001):

i. Utilities of successful and unsuccessful communication of a given proposition

ii. The costs associated with the use of particular sentences

iii. Probabilities of various possible speaker intentions (mutual probabilities)

Games in Extensive Form



Games in Extensive Form

Equilibrium strategies:

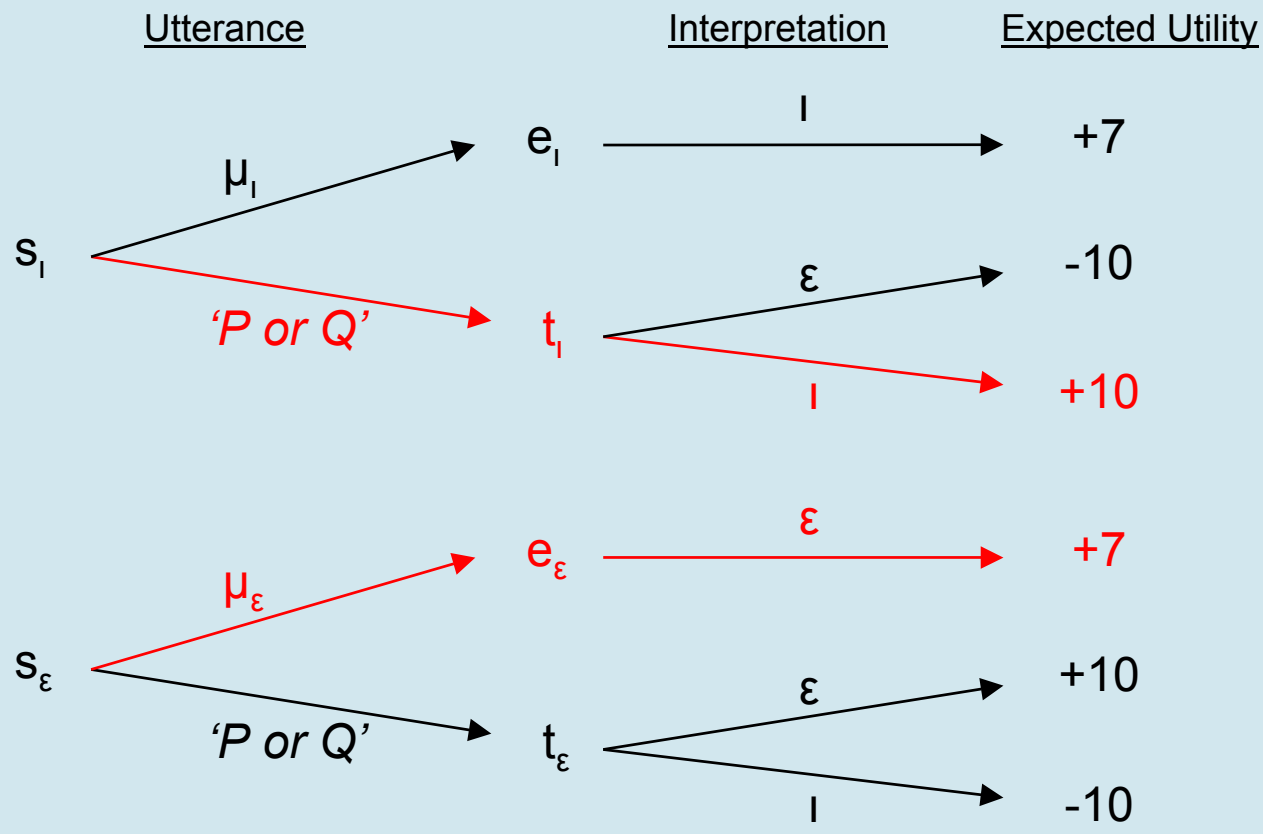
σ_1: 'P or Q' $\rightarrow \iota$

$\mu_\varepsilon \rightarrow \varepsilon$

σ_2: 'P or Q' $\rightarrow \varepsilon$

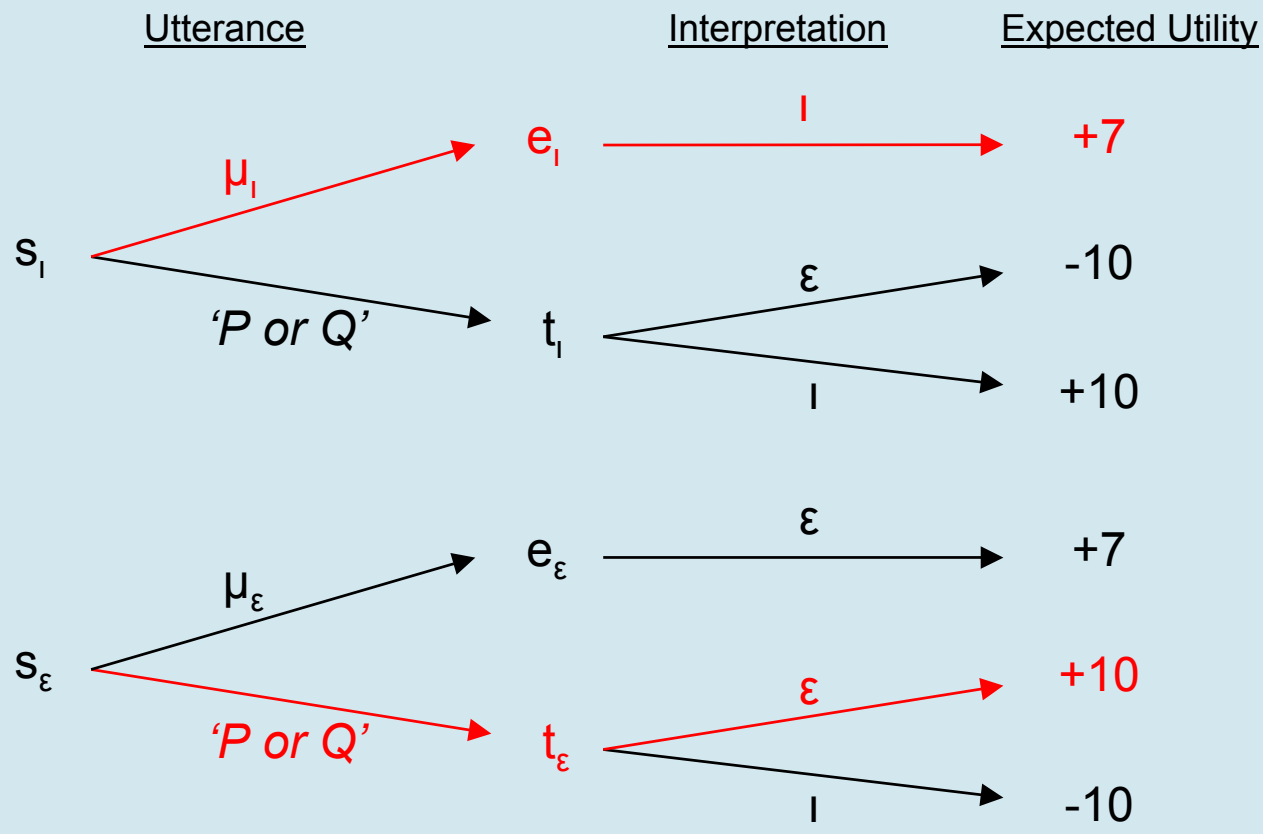
$\mu_\iota \rightarrow \iota$

Games in Extensive Form



$EU(\sigma_1) = P(s_ı)*10 + P(s_ε)*7$

Games in Extensive Form



$$EU(\sigma_2) = P(s_ε)*10 + P(s_ı)*7$$

Games in Extensive Form

Equilibrium strategies:

σ_1: ‘P or Q’ $\rightarrow \iota$

$\mu_\varepsilon \rightarrow \varepsilon$

$EU(\sigma_1) = P(s_\iota)*10 + P(s_\varepsilon)*7$

σ_2: ‘P or Q’ $\rightarrow \varepsilon$

$\mu_\iota \rightarrow \iota$

$EU(\sigma_2) = P(s_\varepsilon)*10 + P(s_\iota)*7$

Games in Extensive Form

- Choice of strategy depends on prior probabilities of possible speaker intentions:

$$P(s_\iota) > P(s_\varepsilon) \rightarrow \quad EU(\sigma_1) > EU(\sigma_2)$$

$$P(s_\iota) < P(s_\varepsilon) \rightarrow \quad EU(\sigma_1) < EU(\sigma_2)$$

Case Study

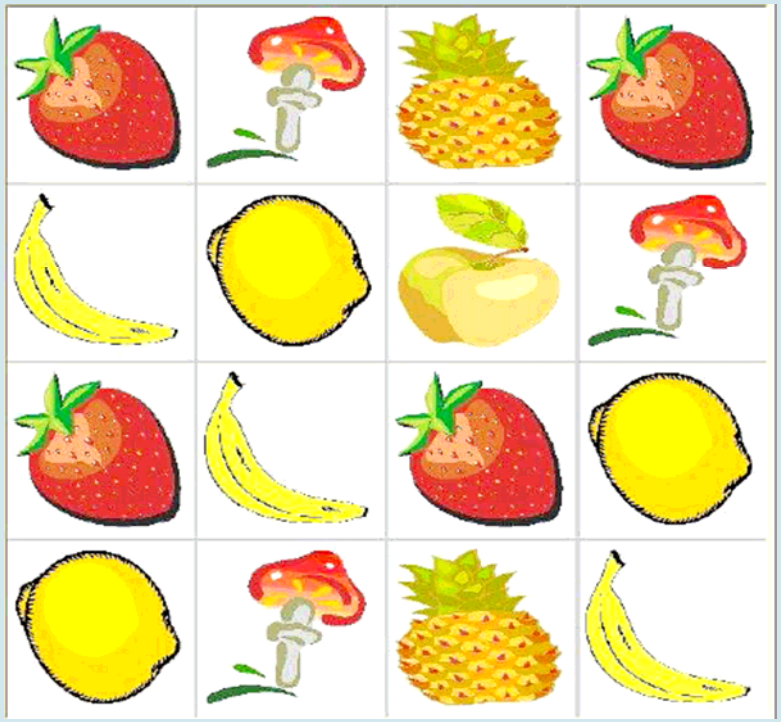
Instantiate model with cooperativity as a variable:

- Speaker cannot or will not use unambiguous alternative.
- Speaker must speak truthfully but stands to gain from miscommunication.

Case Study

(1) *The strawberry is next to a lemon <u>or</u> a pineapple.*

Inclusive (ι): one of *p*, *q* or *pq* holds
Exclusive (ε): one of *p* or *q* holds



p = ‘next to a lemon’

q = ‘next to a pineapple’

Case Study

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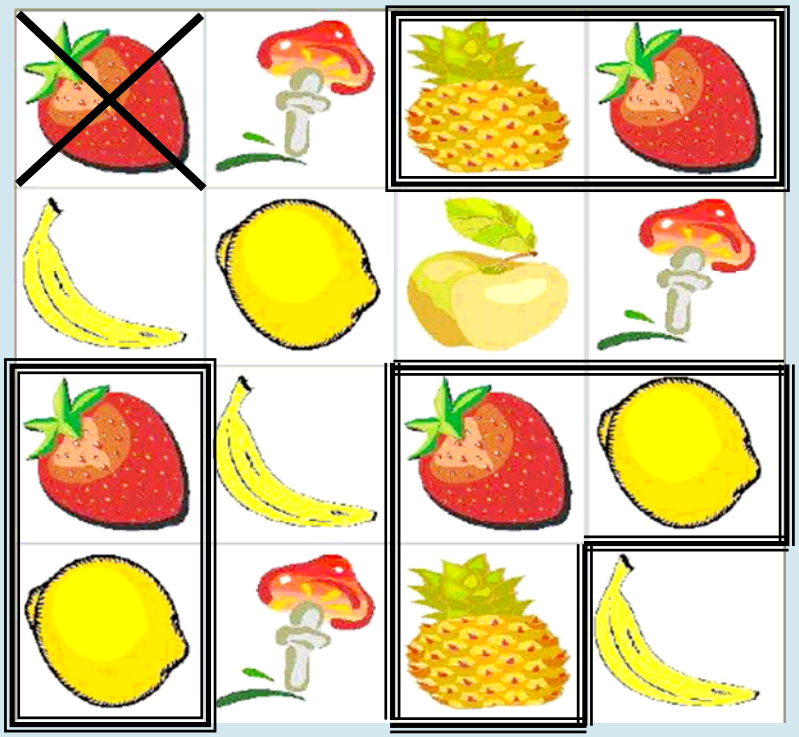

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Case Study

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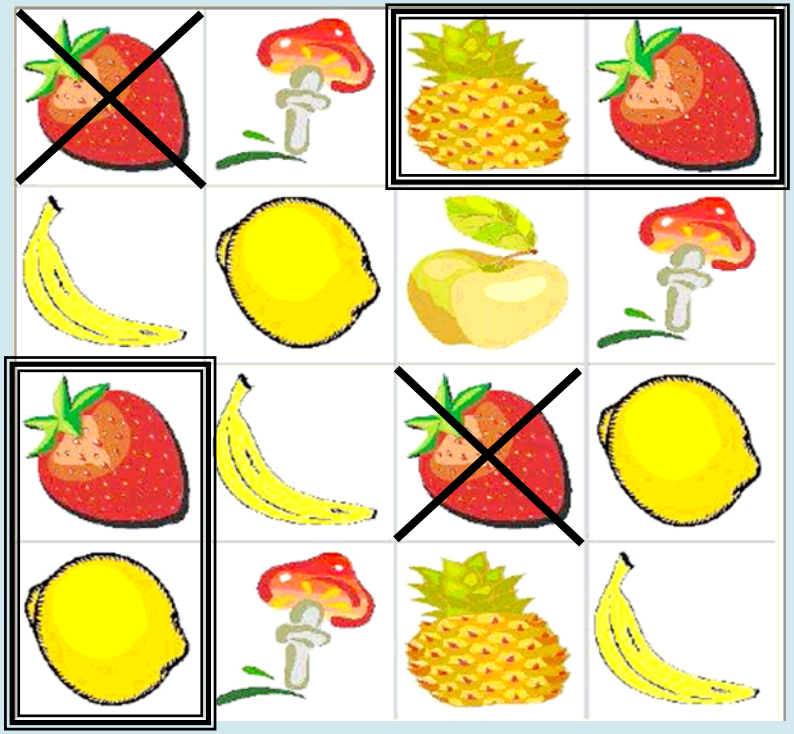


p = ‘next to a lemon’

q = ‘next to a pineapple’

Case Study

Assumptions:

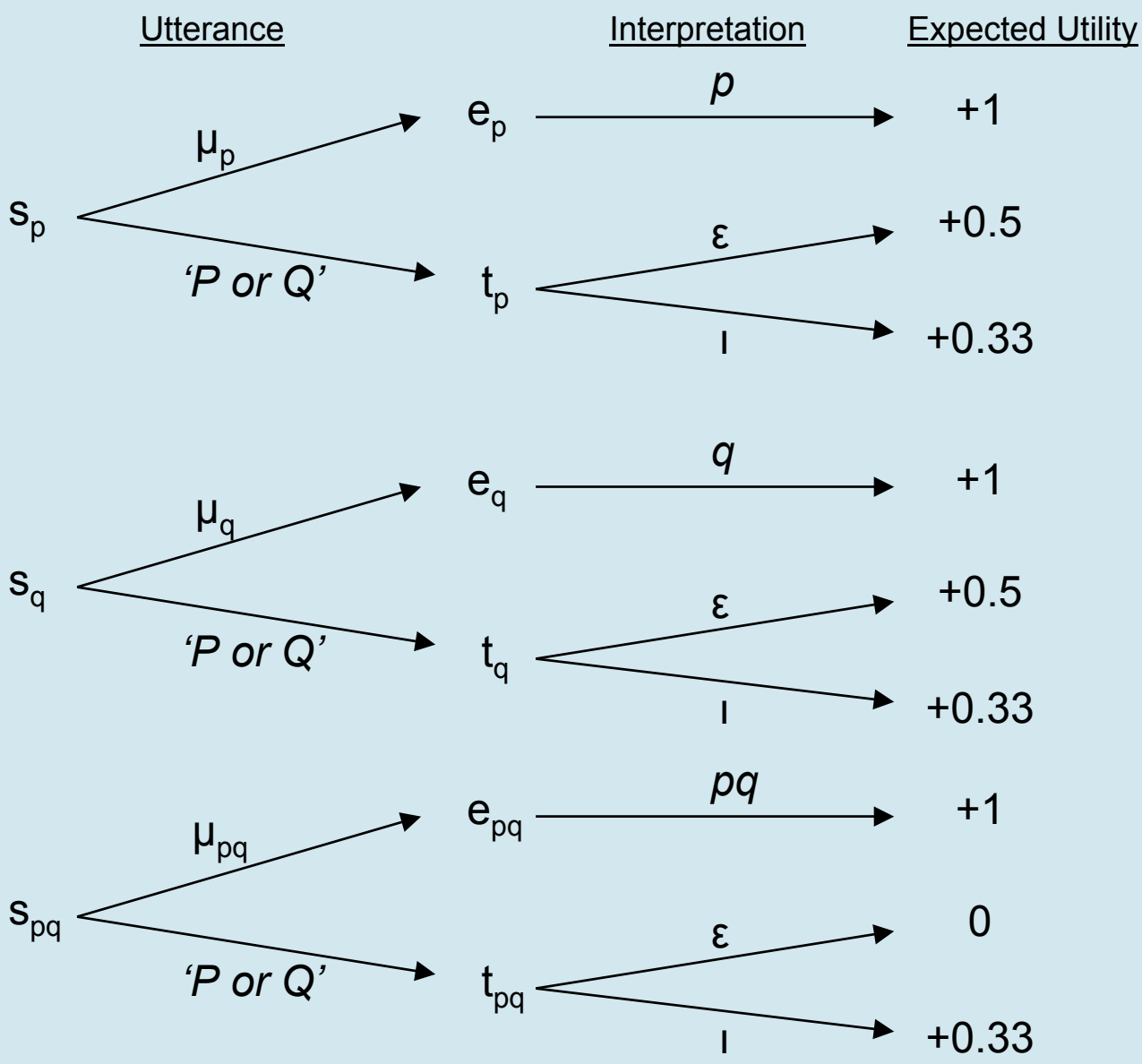
- Mutually believed that speaker knows precisely which one of $p\bar{q}$, $\bar{p}q$, pq, or $\bar{p}\bar{q}$ holds
- Hearer first chooses an interpretation (i.e. a set of strawberries), and then chooses an item from the set at random
- Interpretive choice is whether to strengthen *or* by assuming that speaker is implicating stronger proposition (not semantic disambiguation)
- $\bar{p}\bar{q}$ ruled out following utterance of ‘P or Q’

Case Study

Additional assumptions:

- Payoff is +1 if hearer chooses the (unique) speaker-intended object, 0 otherwise
- μ_p, μ_q, μ_{pq} are unambiguous alternatives to 'P or Q'

  (e.g. 'P and not Q')


Utterance
Interpretation
Expected Utility
s_p
μ_p
'P or Q'
e_p
p
+1
t_p
ε
+0.5
ı
+0.33
s_q
μ_q
'P or Q'
e_q
q
+1
t_q
ε
+0.5
ı
+0.33
s_{pq}
μ_{pq}
'P or Q'
e_{pq}
pq
+1
t_{pq}
ε
0
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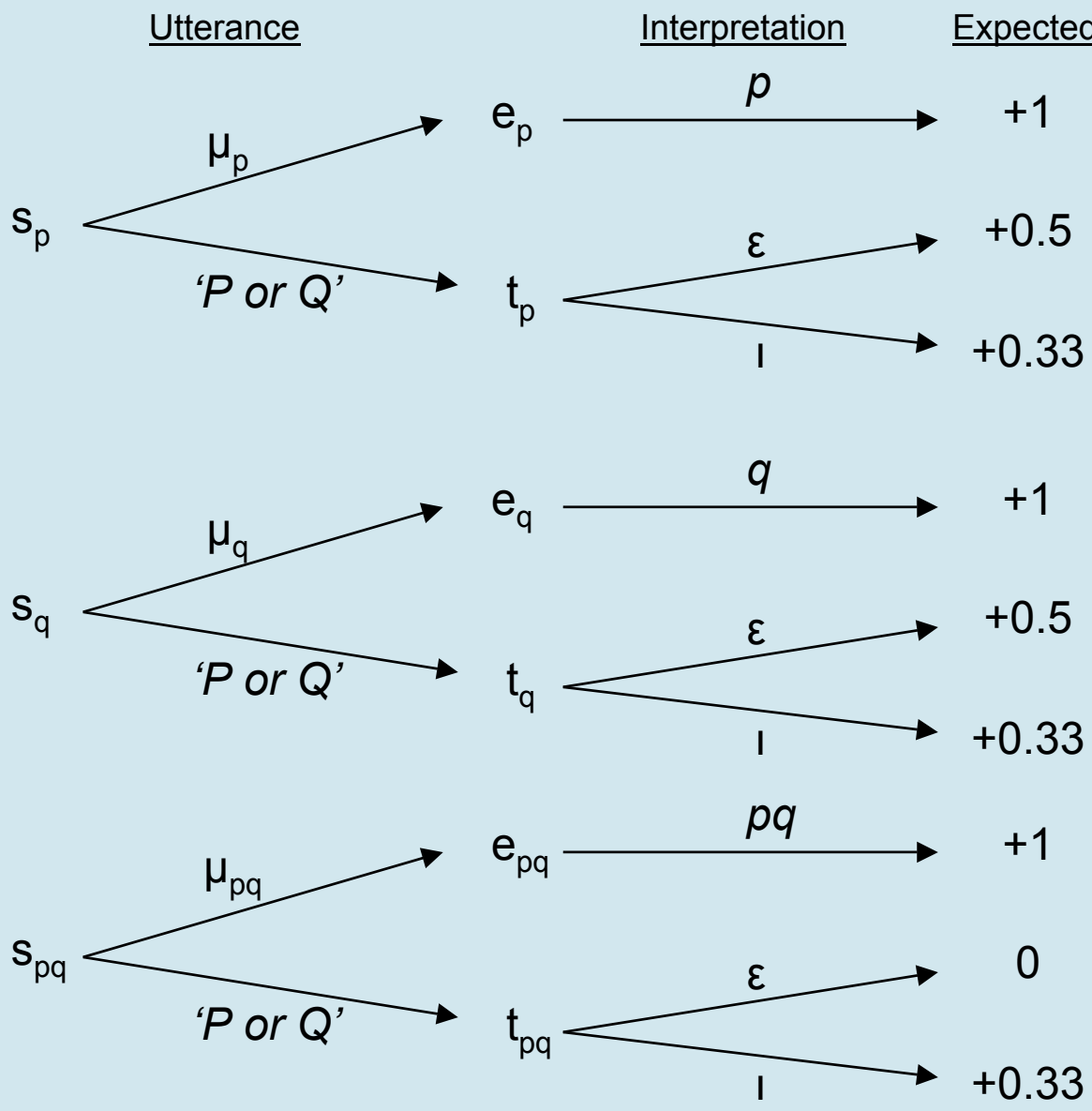
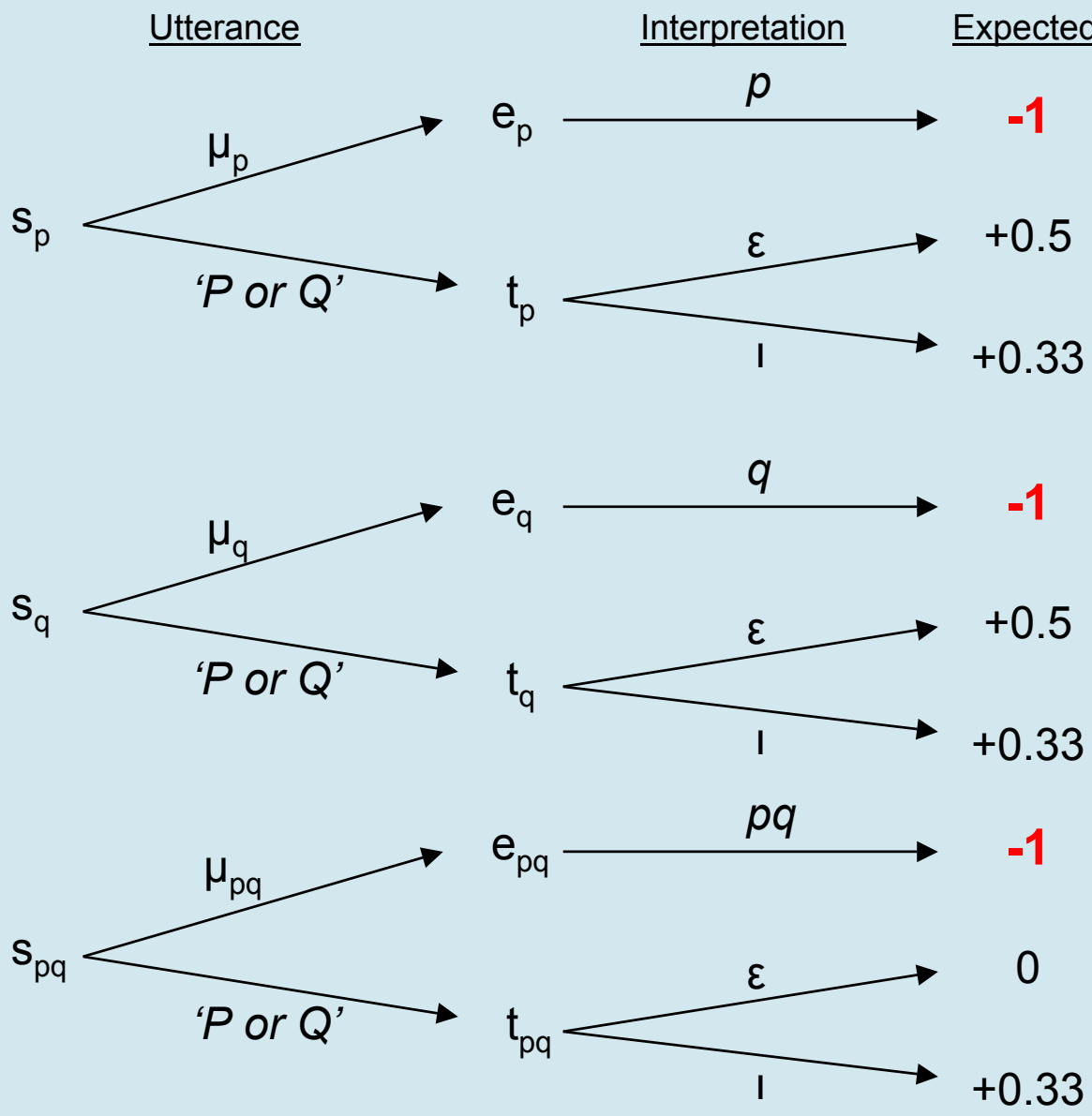
+0.33



$EU(\mu_p) = EU(\mu_q) = EU(\mu_{pq}) = 1$

$EU(ı), EU(ε) < 1$

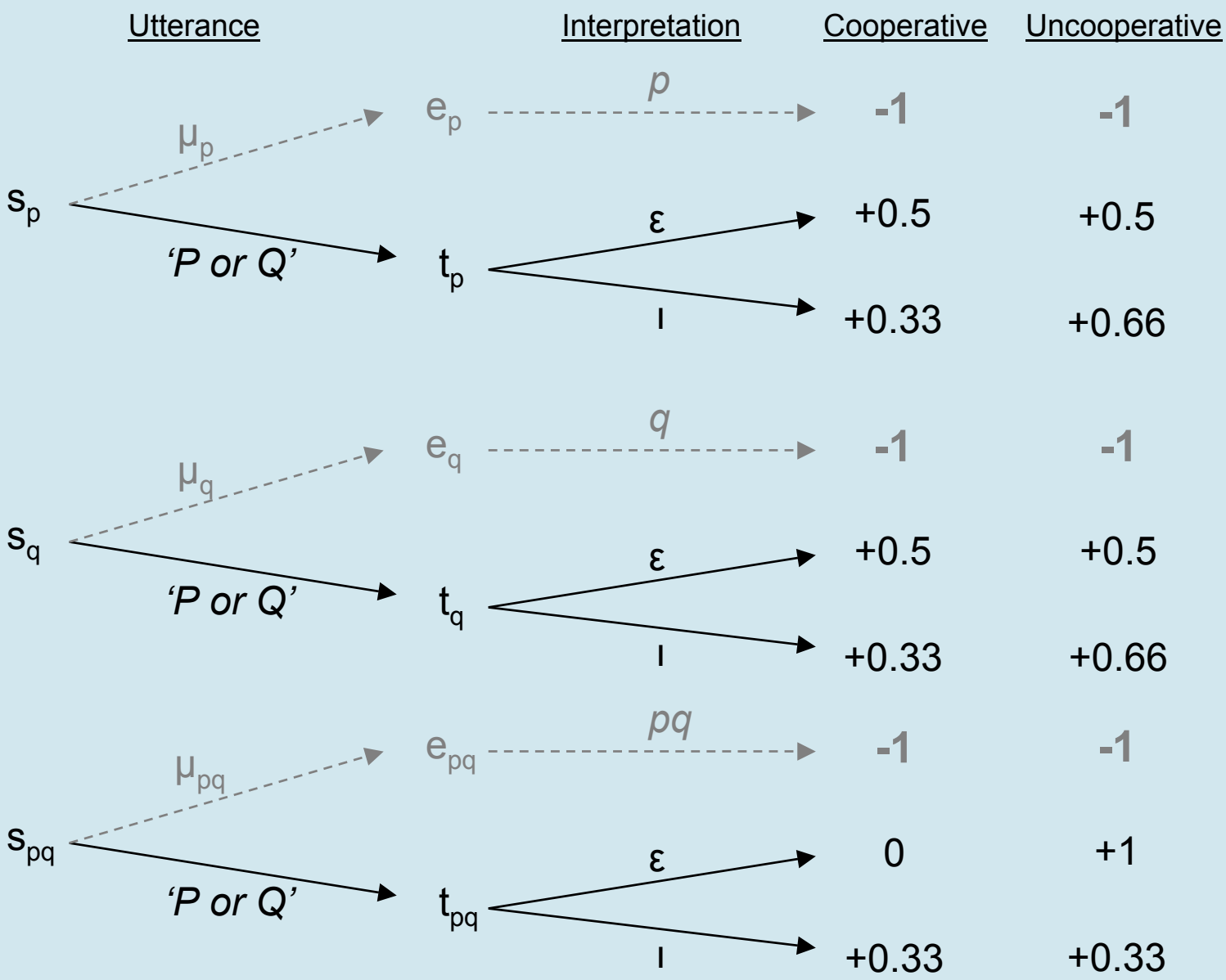
- **Speaker not predicted to use 'P or Q'**



$EU(\mu_p) = EU(\mu_q) = EU(\mu_{pq}) = -1$

$EU(\imath), EU(\varepsilon) = .33$

- **Speaker not predicted to use 'P or Q'**
- **Unless talk is not cheap**


Utterance
Interpretation
Cooperative
Uncooperative
s_p
μ_p
e_p
p
-1
-1
'P or Q'
t_p
ε
+0.5
+0.5
ı
+0.33
+0.66
s_q
μ_q
e_q
q
-1
-1
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ε
+0.5
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+0.33
+0.66
s_{pq}
μ_{pq}
e_{pq}
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-1
'P or Q'
t_{pq}
ε
0
+1
ı
+0.33
+0.33

Case Study

Uniform mutual probability

$P(s_p) = P(s_q) = P(s_{pq}) = .33$

Cooperative:

$EU(\iota) = (.33)*(.33) + (.33)*(.33) + (.33)*(.33)$ **= .33**

$EU(\varepsilon) = (.33)*(.5) + (.33)*(.5) + (.33)*(0)$ **= .33**

Uncooperative:

$EU(\iota) = (.33)*(.66) + (.33)*(.66) + (.33)*(.66)$ **= .66**

$EU(\varepsilon) = (.33)*(.5) + (.33)*(.5) + (.33)*(1)$ **= .66**

EU does not distinguish between strategies.

Case Study

Variable Mutual Probability

$$P(s_p) + P(s_q) + P(s_{pq}) = 1$$

Cooperative:

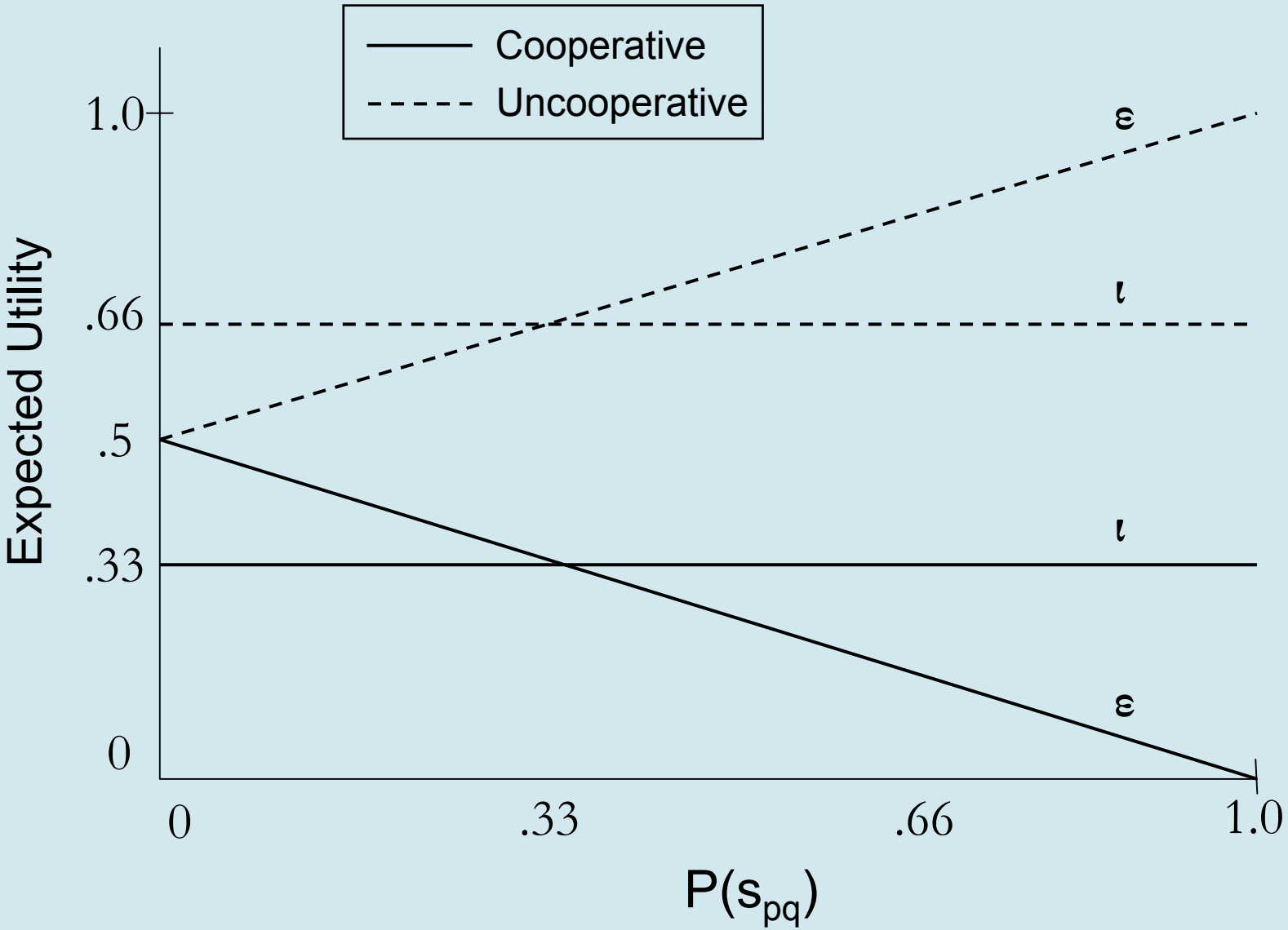
$$EU(\iota) = P(s_p)*(.33) + P(s_q)*(.33) + P(s_{pq})*(.33) = \mathbf{.33}$$

$$EU(\varepsilon) = P(s_p)*(.5) + P(s_q)*(.5) + P(s_{pq})*(0) = \mathbf{.5 - P(s_{pq})*(.5)}$$

Uncooperative :

$$EU(\iota) = P(s_p)*(.66) + P(s_q)*(.66) + P(s_{pq})*(.66) = \mathbf{.66}$$

$$EU(\varepsilon) = P(s_p)*(.5) + P(s_q)*(.5) + P(s_{pq})*(1) = \mathbf{P(s_{pq})*(.5) + .5}$$


Cooperative
Uncooperative
1.0
.66
.5
.33
0
Expected Utility
ε
ι
ι
ε
0
.33
.66
1.0
P(s_pq)

Case Study

Variable Mutual Probability

- Preferred strategy depends on both cooperativity and mutual probability.

	Cooperative	**Uncooperative**
$P(s_{pq}) < .33$	$EU(\iota) < EU(\varepsilon)$	$EU(\iota) > EU(\varepsilon)$
$P(s_{pq}) > .33$	$EU(\iota) > EU(\varepsilon)$	$EU(\iota) < EU(\varepsilon)$
$P(s_{pq}) = .33$	$EU(\iota) = EU(\varepsilon)$	$EU(\iota) = EU(\varepsilon)$

Case Study

Features of the model:

- Interpretations are measurable.
    - Object selection is constrained by interpretation.
    - Aggregate distribution of hearer's choices indicates interpretative choices.
- Payoff structure is controlled.
    - EU of interpretative strategies is determined directly by payoffs assigned to concrete outcomes.
- Interpretive strategy is predicted to vary with speaker's payoff structure (i.e. cooperativity).

Experiment

Cooperativity as a variable

- Does the interpretation of logical connectives vary under systematic manipulation of the speaker's payoff structure?

Experiment - Overview

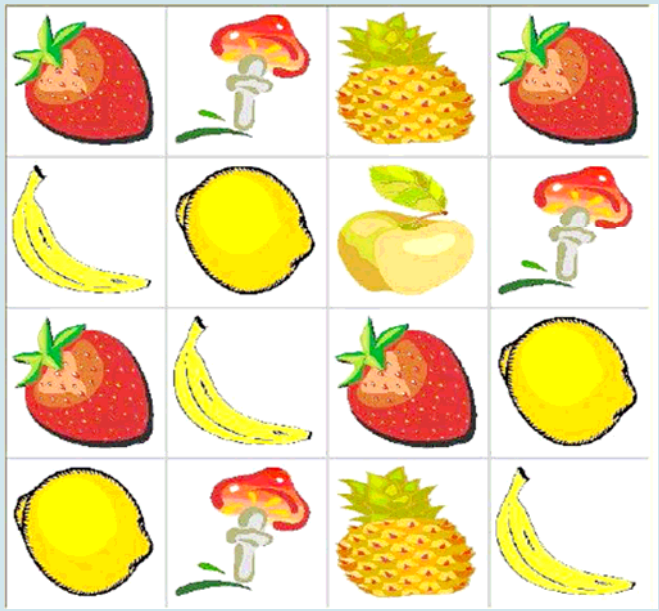
- Two fictional characters playing a series of games.
- Participant's task: find a coin that the players have hidden behind one of the objects in a grid.
- One player provides a verbal clue:
    - Must tell the truth
    - May not give away the location completely
- One player wins the game only if the participant chooses correctly, and the other wins only if the participant chooses incorrectly.
- Vary whether clue-giver stands to win or lose.

Experiment – Materials and Procedures

Test items:

- 4x4 object grid
- Verbal ‘clue’
- Selection of object with mouse

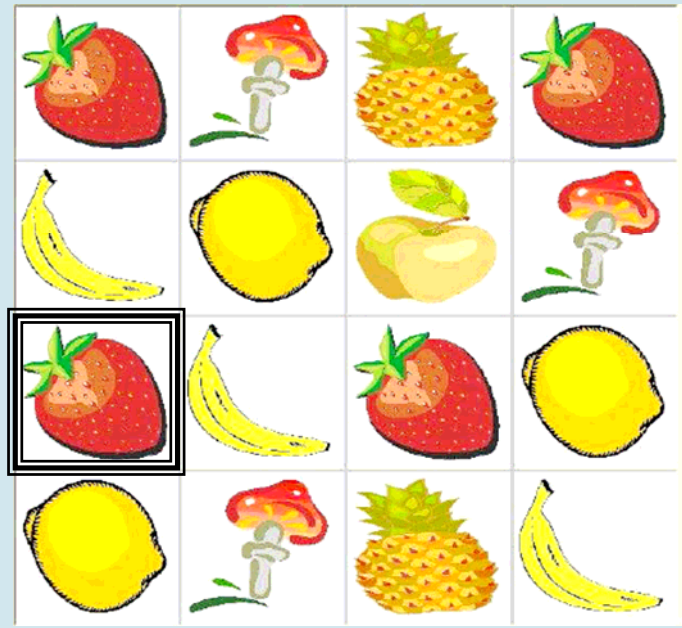
Sandra says, “The prize is behind a strawberry that is next to a lemon or a pineapple.”



Experiment – Materials and Procedures

Four potential targets per grid:

- Next to P only
- Next to Q only
- Next to P and Q
- Next to neither P nor Q



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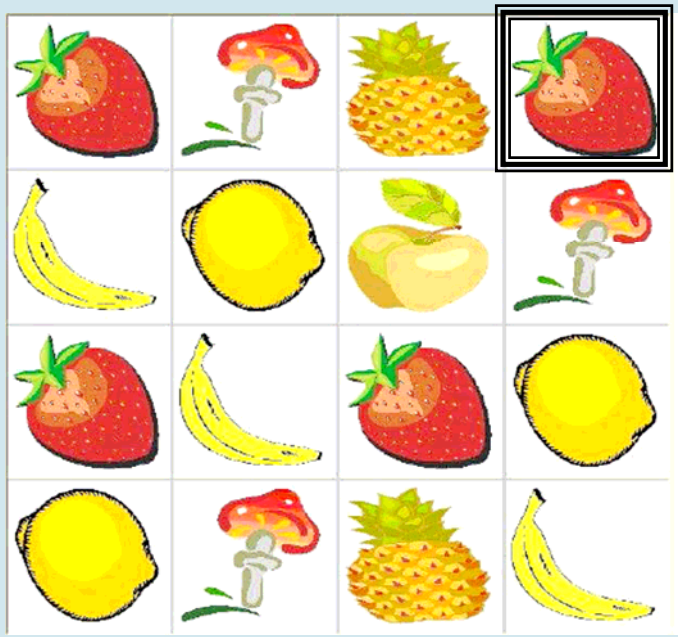

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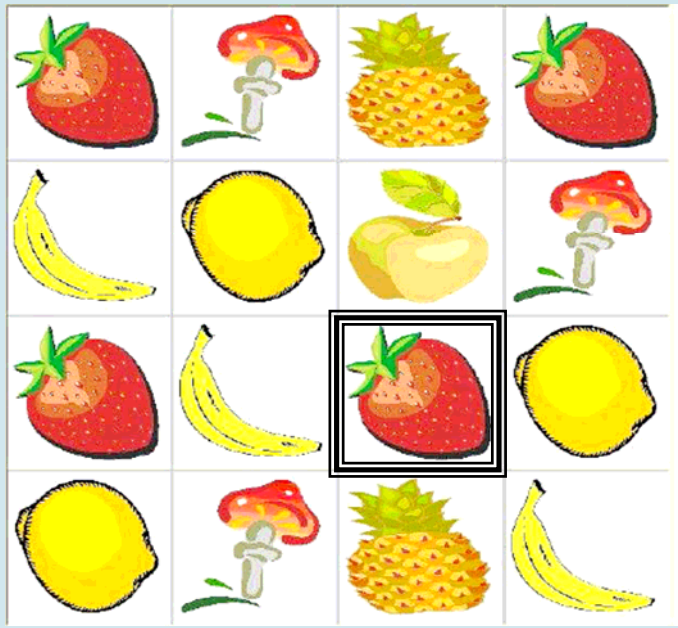

Experiment – Materials and Procedures

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- Next to P only
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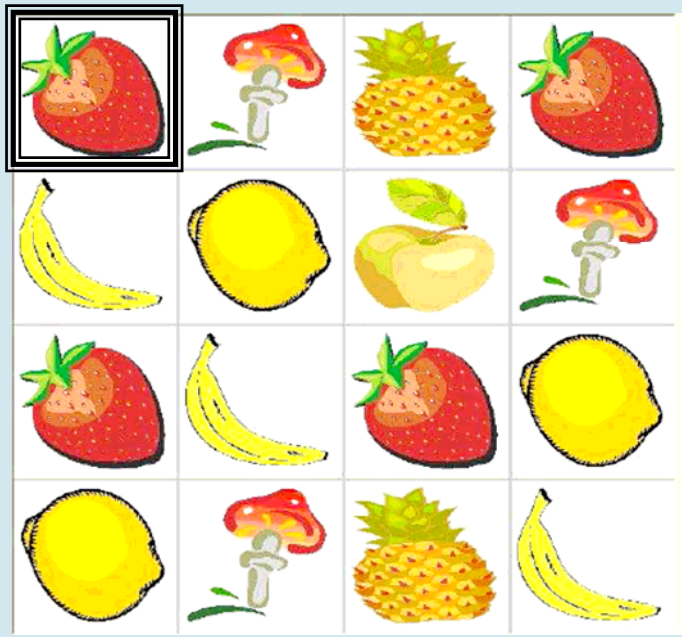


Experiment – Materials and Procedures

- 3 clue-types:

*The prize is behind a strawberry that is next to a lemon **or** a pineapple.*

*The prize is behind a strawberry that is **not** next to a lemon **and** a pineapple.*

*The prize is behind a strawberry that is next to a lemon **if** it is next to a pineapple.*

Experiment – Materials and Procedures

- Block = 3 clue types x 7 items = 21 items
- 4 blocks = 84 items total
- Clue type pseudorandomized within blocks
- Speaker interest (i.e., cooperative vs. uncooperative) alternated by block
- Block order rotated

Experiment – Materials and Procedures

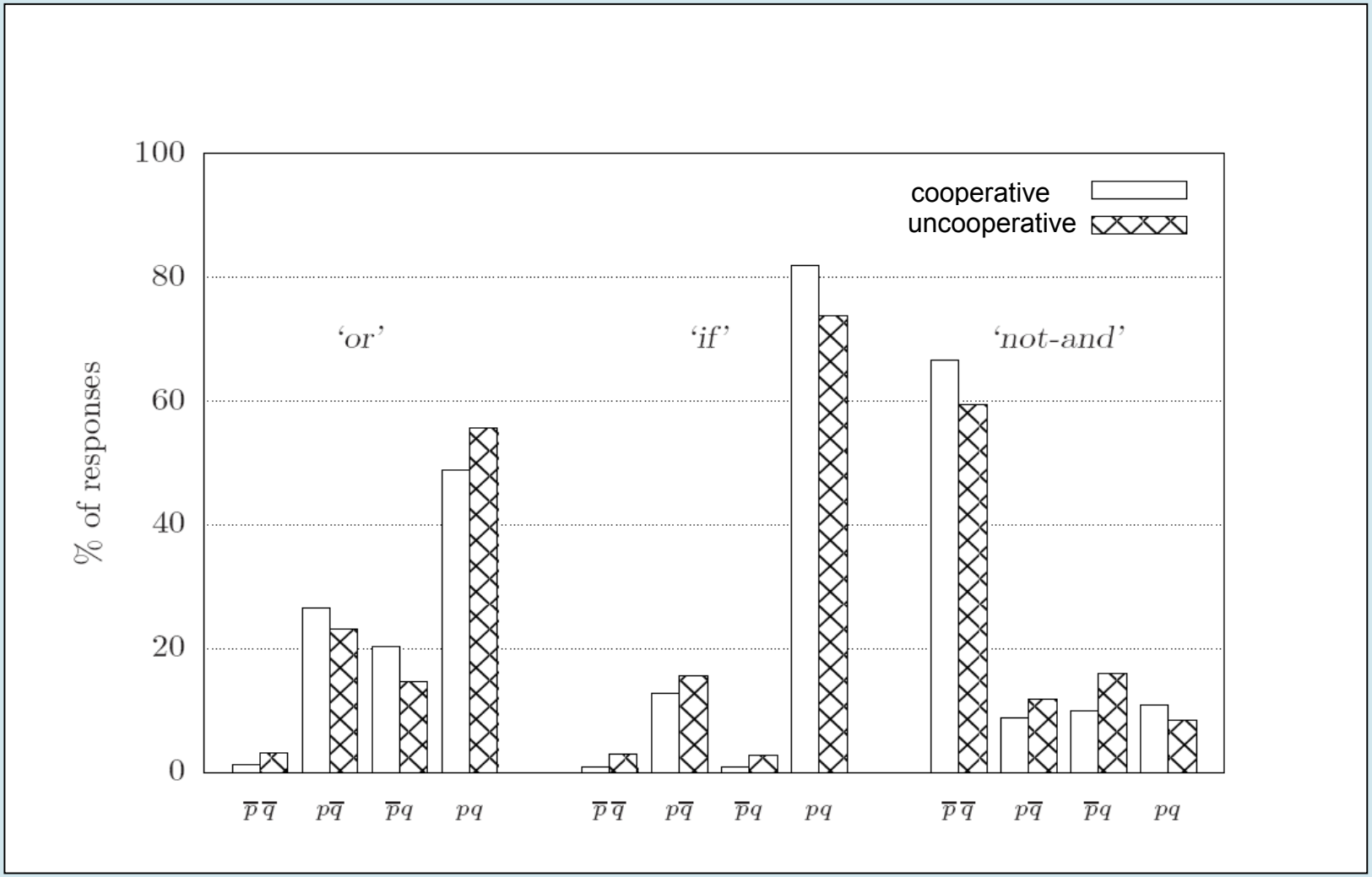
- No time limit
- No feedback
- Coded for target type selected

  (i.e. A only, B only, A and B, neither)

Experiment - Subjects

- 15 undergraduate students, Northwestern U.
- Introductory linguistics courses
- Course credit
- Native, 1st language English speakers

Experiment - Results



Experiment – Results

- Significant interaction (2-way ANOVA): sentence-type, speaker role, response pattern
- Significant interaction (2-way ANOVA): sentence-type, response pattern
- Significant main effect: response pattern
- *if-then* (planned 2-way ANOVA): speaker role, response pattern

Experiment – Results (*or*)

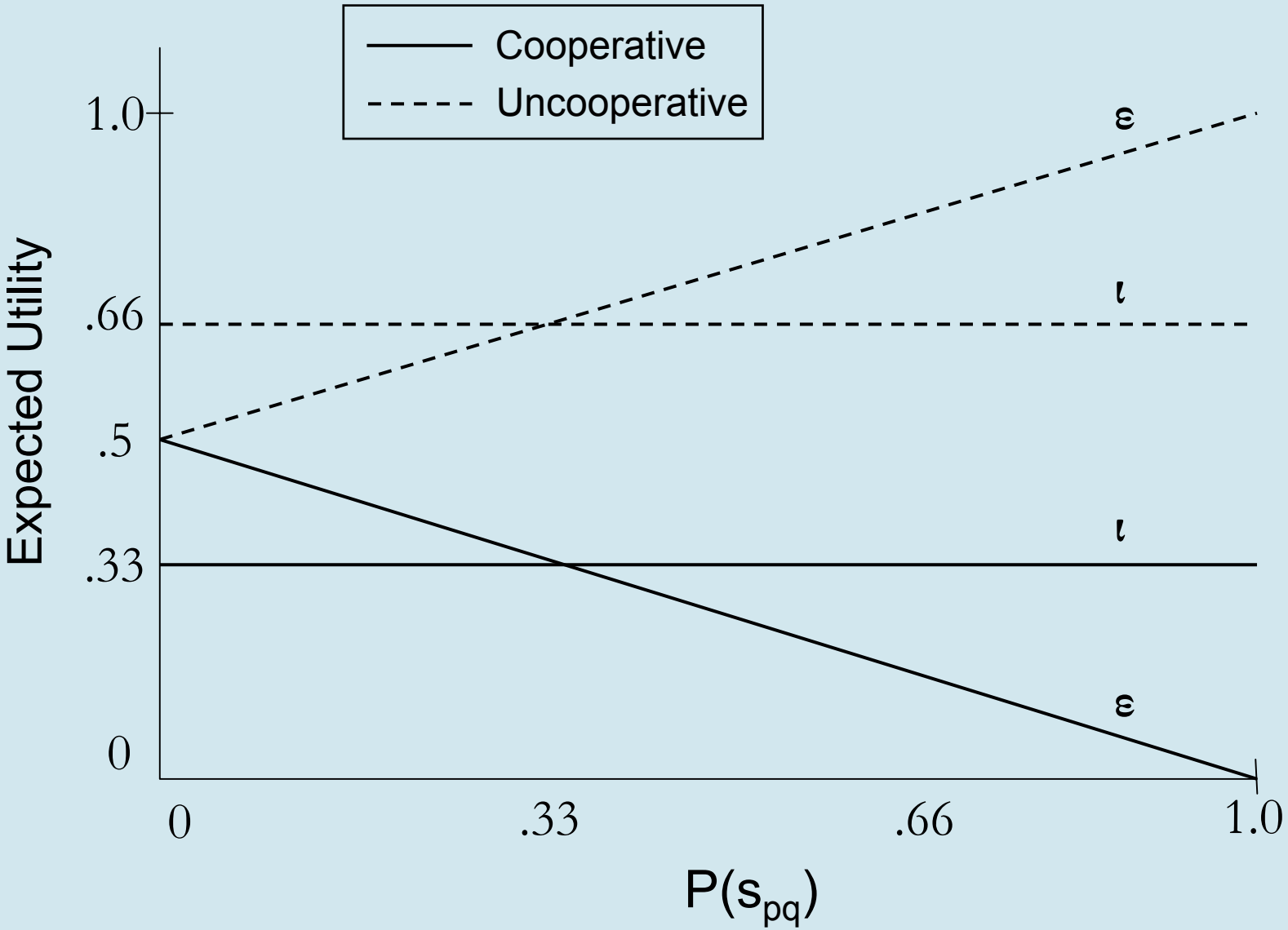
***pq* response in ≈50% of trials**

- Consistent with inclusive interpretation
- Predicted by model only when P(pq) is high (>.33) for cooperative speaker and low (<.33) for uncooperative speaker
- Overall result suggests that P(pq) is high

Experiment – Results (*or*)

Cooperativity effect

- Exclusive interpretation more often with cooperative speakers (not significant)
- Expectation that uncooperative speakers resort to unconventional use of *or* when trying to deceive
- Suggests P(pq) low (<.33)


Cooperative
Uncooperative
1.0
.66
.5
.33
0
Expected Utility
ε
ι
ι
ε
0
.33
.66
1.0
P(s_pq)

Experiment – Results (*if-then*)

pq response in >70% of trials

- Consistent with indirectness condition
  - Not predicted to arise when speaker has full knowledge; *if-then* predicted to reduce to $\rightarrow$.

    $(q \vee \sim p) \wedge \sim(q \wedge \sim p)$
  - According to results for *or*, predict $p\bar{q}$
  - Inference: speaker has evidence for high probability of co-occurrence

Experiment – Results (*if-then*)

Cooperativity effect

- pq (i.e. pragmatic interpretation) more likely with cooperative speaker
- Expectation that uncooperative speakers resort to unconventional use of *if*-then when trying to deceive

Experiment – Results (*not-and*)

$\bar{p}\bar{q}$ response in >60% of trials

- Consistent with pragmatic strengthening

$$\sim(p \wedge q) \wedge \sim(p \vee q)$$

Experiment – Results (*not-and*)

Cooperativity effect

- Pragmatic interpretation more likely with cooperative speaker (not significant)
- Expectation that uncooperative speakers resort to unconventional use of *if*-then when trying to deceive
- Rate of inappropriate responses (pq) ≈ rate of pragmatic-appropriate responses ($p\bar{q}$, $\bar{p}q$)
- Maybe some trouble interpreting operator

Experiment – Results (*Overall*)

All three sentence types:

- One possibility chosen over other two
- Participants assign a high probability to one possibility
- No such bias introduced by experiment

Conclusion:

- Bias arises from the speaker's use of the sentence
- Hearers drawn toward object that is next to both mentioned (except for *not-and*)

Experiment – Conclusions

- Cooperativity influences the likelihood of a pragmatically stronger interpretation
- Pragmatic strengthening more likely with cooperative speakers

Future Directions

- More realistic speech situation
- Other Gricean variables
    - Mutual probabilities
    - Rationality

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Grice on conditionals

If *A*, then *C*

- Truth conditions: Material Conditional $A \rightarrow C$
- Implicature: Indirectness Condition

> "There are non-truth-functional grounds for accepting $A \rightarrow C$."

  - The acceptance of $A \rightarrow C$ is not solely based on the belief in $\neg A$ or in C.
  - If it were, then the speaker could – hence should – have used a simpler form.

- Question: What are those non-truth-functional grounds?

Grice on conditionals

If *A*, then *C*

- The point of using conditionals:
  - to assure the hearer that *Modus Ponens* holds.
  - "Should you learn *A*, it will be safe to infer *C*"

- Jackson's (1979, 1987) elaboration:
  - *Robustness*: The conditional will remain likely if *A* turns out true.
  - *Probability*: The conditional is *robust* iff (or to the extent that) the conditional probability *P(C|A)* is high.

Grice on conditionals

- Cancellability of the Indirectness Condition

  Suppose that I am propounding ... a puzzle ... about people whom I can see but my hearer cannot. I could perfectly properly say, at some point,

  - ``If Jones has black (pieces) then Mrs. Jones has black too."

  To say this would certainly not be to implicate the fulfillment of the Indirectness Condition; indeed, the total content of this utterance would be just what would be asserted (according to truth table definition) by saying

  - ``Jones has black → Mrs. Jones has black."

- When the speaker full knowledge of the relevant facts:
  - the Indirectness Condition does not arise;
  - `If *A*, then *C*' is interpreted as $A \rightarrow C$.

  This is what's predicted to happen in our experiment.