Logical Consequence: From Logical Terms to Semantic Constraints

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The Concept of Logical Consequence [Tarski, 1936]

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Criteria for logical terms: [Peacocke, 1976, McCarthy, 1981, Sher, 1991, McGee, 1996, Feferman, 1999, Bonnay, 2008].

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There are: [Fox, 2000, Gajewski, 2002, Fox and Hackl, 2006]

Overview

The Thesis of the Centrality of Logical Terms

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- (TF2) The form of a sentence is determined by the logical vocabulary and the arrangement of all terms in the sentence.

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The logical validity of an argument is determined by the logical vocabulary and the arrangement of all terms in the sentences of the argument.

- (PD) There is a principled distinction between logical and nonlogical terms.
- (TR) Logical validity is relative to a choice of logical terms, and there is no principled distinction between logical and nonlogical terms.

Motivation: Form and What is Fixed

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- TF2 is motivated by the idea that form has to do with what is fixed.
- There may be different reasons for holding some things fixed and others variable.
- These reasons still do not warrant the strict dichotomy between logical and nonlogical terms.

Semantic Constraints

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$$(\wedge)\text{: } I(\varphi \wedge \psi) = T \Leftrightarrow I(\varphi) = T \text{ and } I(\psi) = T$$

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Semantic Constraints

Fixing something amounts to limiting the admissible interpretations.

$$I(allRed) \cap I(allGreen) = \emptyset$$

- $I(even) \cap I(odd) = \emptyset$
- $I(bachelor) \subseteq I(unmarried)$
- $I(H_2O) = I(water)$
- $I(wasBought) = I(wasSold)$
- $I(\exists) = \{A \subseteq D : A \neq \emptyset\}$
- $0 \in I(naturalNumber)$
- $I(prime) = \{2, 3, 5, ...\}$
- $I(P) \subseteq D$
- $I(John) \in D$
- $I(s) = T$ or $I(s) = F$

- $I(R)$ is a symmetric binary relation.
- $I(abc)$ is a sentence.
- $I(d) \neq I(\wedge)$
- $I(or) \in \{f_{\vee}, f_{\underline{\vee}}\}$ where $f_{\vee}$ is the inclusive or function, and $f_{\underline{\vee}}$ is the xor function from pairs of truth values to truth values.
- $I(Q) = \{A \subseteq D : 0 \in A\}$ (Q is a nonstandard quantifier.)

The language and its models

Language

- Terms
- Phrases

Models

$M = \langle D, I \rangle$

- D (the domain) is a non-empty set.
- I (the interpretation function) assigns values to phrases from the set-theoretic hierarchy with $D \cup \{T, F\}$ as ur-elements.

Logical Consequence

Let Δ be a set of semantic constraints, such as those mentioned above. A *Δ-model* is an *admissible model* by Δ, i.e. a model abiding by the constraints in Δ.
An argument $\langle \Gamma, \varphi \rangle$ is *Δ-valid* ($\Gamma \models_\Delta \varphi$) if for every Δ-model M, if all the sentences in Γ are true in M, then φ is true in M.

So, for instance we have:

$bachelor(John) \models_\Delta unmarried(John)$.

Determinacy

A term a is *determined by the set of terms B* (w.r.t. Δ) if for any two Δ-models $M = \langle D, I \rangle$ and $M' = \langle D', I' \rangle$, if $I(b) = I'(b)$ for all $b \in B$ then $I(a) = I'(a)$.

Determinacy

For convenience, we treat the domain as a term, that is, add a (pseudo)-term $\mathcal{D}$ and a constraint:

- $I(\mathcal{D}) = D$

Logical Terms

We can define the *logical terms* (the "completely fixed" terms) of the system as those terms that are determined by the domain, i.e. by $\{\mathcal{D}\}$.

Compositionality and Extensionality

A language L is *compositional* (w.r.t. Δ) if each phrase p consisting of the terms $a_1, ..., a_n$ and auxiliary symbols is determined by $\{\mathcal{D}, a_1, ..., a_n\}$ (w.r.t. Δ).

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Remark. Let L be a language that is compositional w.r.t. a set of semantic constraints Δ. A term a in L is a logical term w.r.t. Δ iff any phrase p consisting of the terms $a_1, ..., a_n$ and auxiliary symbols is determined by $\{a_i : 1 \leq i \leq n, a_i \neq a\} \cup \{\mathcal{D}\}$.

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A language L is *extensional* (w.r.t. Δ) if every sentence consisting of the terms $a_1, ..., a_n$ and auxiliary symbols is determined by $\{\mathcal{D}, a_1, ..., a_n\}$ (w.r.t. Δ).

Dependency

A set of phrases A *depends* on the set of phrases B (w.r.t. Δ) if there are Δ-models $M = \langle D, I \rangle$ and $M' = \langle D, I' \rangle$ sharing the same domain D such that for any Δ-model $M^* = \langle D, I^* \rangle$, if $I^*(b) = I(b)$ for all $b \in B$, then $I^*(a) \neq I'(a)$ for some $a \in A$ (that is, fixing the phrases in B in a certain way excludes some interpretation for the phrases in A that can otherwise be realized).

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Example: by the constraint $I(bachelor) \subseteq I(unmarried)$, $\{bachelor\}$ depends on $\{unmarried\}$: let $I(unmarried) = \{John, Mary\}$, $I'(bachelor) = \{John, Jim\}$, so for any I^* such that $I^*(unmarried) = I(unmarried)$, $I^*(bachelor) \neq I'(bachelor)$.

Determinacy and Dependency

Proposition. For every term a and set of terms B:

1. If a is a logical term, then a is independent of B.
2. If a is determined by B, and a is not a logical term, then a depends on B.

Failure of Substitution?

$$\neg\exists x(allRed(x) \wedge allGreen(x))$$

is valid, but

$$\neg\exists x(even(x) \wedge prime(x))$$

is not.

$\forall$	water	allRed	John	big
$\exists$	H_2O	allGreen	Gila	thinks
$\neg$	wasBought	allYellow	Alfred	number
$\wedge$	wasSold	allBlue	Rudolf	fast

Induced Permutations

Let π be a permutation on the terms of L.

π can be extended to the phrases of L.

π can be further extended to apply to models:
For $M = \langle D, I\rangle$, $\pi(M) = \langle D, I^*\rangle$ where for each phrase s,
$I^*(s) = I(\pi(s))$.

Category

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A set of terms A is a *category* (w.r.t. Δ) if every two terms in A are interchangeable, and no term $a \in A$ is interchangeable with a term $b \notin A$.

$\forall$	water	allRed	John	big
$\exists$	H_2O	allGreen	Gila	thinks
$\neg$	wasBought	allYellow	Alfred	number
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Schema

A *schema* is a string of schematic letters such that each schematic letter is assigned to a category.

$\neg\exists x(\mathit{allRed}(x) \wedge \mathit{allGreen}(x)) \mapsto \hat{\neg}\hat{\exists} x(\mathcal{R}(x)\hat{\wedge}\mathcal{G}(x))$

Schema

A schema is *valid* if all its instances are valid. A schema is *invalid* if all its instances are not valid.

Substitution Restored

Proposition. Every schema is either valid or invalid.

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Corollary. A sentence is valid iff it is an instance of a valid schema.

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- The blended approach, revised: Models represent possible worlds under reinterpretations admissible by the set of semantic constraints.
  - Semantic constraints can be thought of as commitments made by reasoners with respect to language.
- The semanticist's/epistemic approach [Zimmermann, 1999]: The range of models describes the semanticist's information

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Conclusion

- Both principled and relativistic accounts of logical terms presuppose the thesis of the centrality of logical terms.
- Understanding *form* as *what is fixed* does not entail a strict dichotomy between logical and nonlogical terms.
- *Semantic constraints* provide various ways of fixing things in the language, that are not limited to the logical/nonlogical distinction.
- The question of a *principled* distinction between logical and nonlogical terms turns into the question: *are there "correct" semantic constraints for logical consequence?*

Thank You!

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