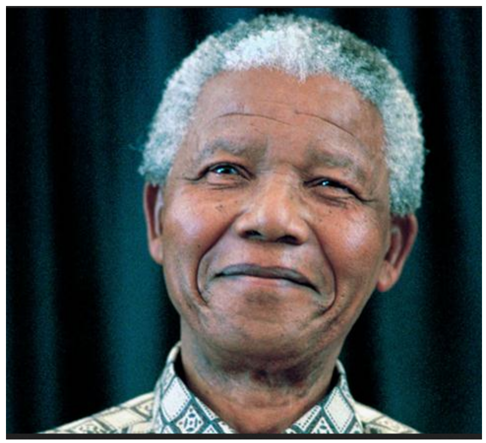
What Kind of Theory Is a Model-Theoretic Semantics of a Natural Language?

Stanley Peters
Stanford University

What do these three have in common?

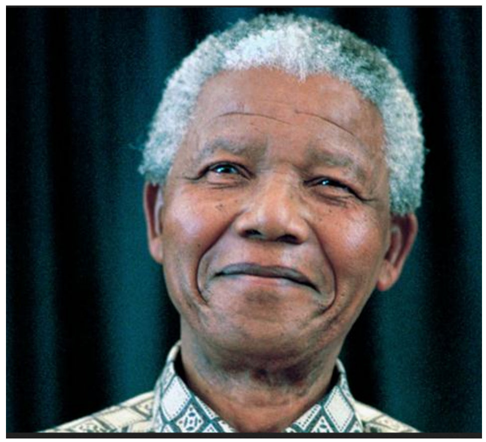


What do these three have in common?

They are all models!

Gisele Bündchen

a fashion model



Nelson Mandela

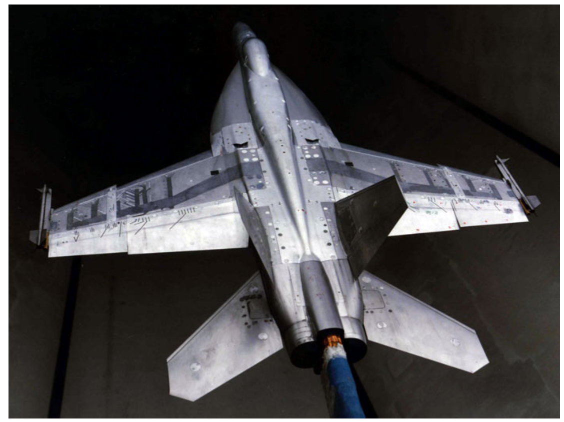
a model statesman

Museum display

a model of Göttingen ca. 1890

Models of something or other

Model of Göttingen ca. 1890



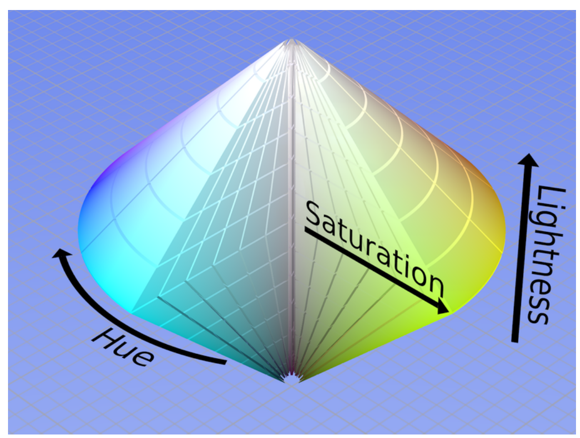
Wind tunnel model of F-18

$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt}$$

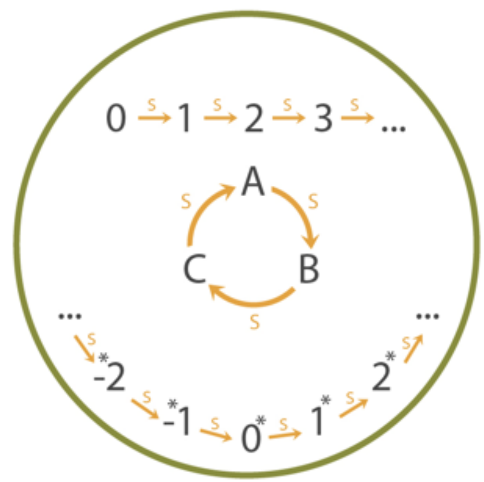
$$C_{A0} - C_A + r_A t = t\,\frac{dC_A}{dt}$$

Numerical model of chemical reactor startup

Mouse model of disease



Model of colors



Non-standard model of natural numbers

A Model's Purpose

- Represent spatial relationships

- Measurement of aerodynamic properties

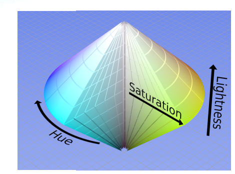
- Calculate time to steady state of a mix-reactor

$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt}$$

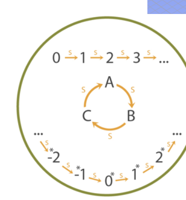
$$C_A + r_A t = t\frac{dC_A}{dt}$$

- Determine causes, development of, and effectiveness of treatments for human diseases

- Spatially represent differences in color characteristics



- Demonstrate that Peano's Axioms do not characterize the natural numbers exactly



What Makes Something a Model?

- A model's *construal* (which depends on its purpose and is usually not a theory) determines how **faithful** it is to what it models. The model's *structure* determines its significant **theoretical characteristics**.
- The substance of a model usually has no significance! It can be chosen to be suitable for the model's purpose.
  - sets
  - vector spaces
  - canonical proofs
- We analyze faithful models as a way of studying certain properties of what they model.
- Ease of analyzing a model's structure can be in tension with transparency of the model's construal.

Is the argument valid?

$(x)\ [like'(x,m) \rightarrow x = c]$

____

$\neg\, like'(p,m)$

IFF given another proposition!

$(x)\ [like'(x,m) \rightarrow x = c]$

$\neg c = p$

____

$\therefore\ \neg like'(p,m)$

Distinct elements of a model's domain must be construed as different entities!

$(x)\ [like'(x,m) \rightarrow x = c]$

$\neg c = p$

____

$\therefore\ \neg like'(p,m)$

$\exists x \exists y\ [\neg x = y\ \&\ P(x)\ \&\ P(y)]$

Why Formalize?

- To clarify, assess consistency, rigorously define predictions/explanations, limit construals/scope of application

  Just as important for language as in physical sciences!
- Intuitive semantics

  $\Downarrow$
- Proof theory

  $\Downarrow$
- Model theory

Why model theory for natural languages?

Logical truth and logical implication

- Twas brillig, and the slithy toves
  Did gyre and gimble in the wabe

  Class of logically possible models

Why the semantics of generalized quantifiers requires model theory

- most(A,B)
- $|A \cap B| > |A - B|$

Why the semantics of generalized quantifiers requires model theory

- most(A,B)

- $|A| \leq k$ & $|A \cap B| > |A - B|$

Why the semantics of generalized quantifiers requires model theory

- most(A,B)

$$|A \cap B| > |A - B|$$

Why the semantics of generalized quantifiers requires model theory

- most(A,B)

- $|A| \leq k$ & $|A \cap B| > |A - B|$

Why the semantics of generalized quantifiers requires model theory

- most(A,B)
- $|A \cap B| > |A - B|$ when $|A| \leq k$
- $\bot$ when $|A| > k$

fewer than zero(A,B)

Why the semantics of generalized quantifiers requires model theory

- most(A,B)
- $|A \cap B| > |A - B|$

So model theory may sometimes fade into the background of natural language semantics, but it is still importantly there.

Why model theory for natural languages?

Logical truth and logical implication

- Twas brillig, and the slithy toves
  Did gyre and gimble in the wabe

  Class of logically possible models

Analytic truth and entailment

- Twas twilight, and the hungry calves
  Did call and frolic in the field
- Twas windy, and the foamy waves
  Did crest and break in the boat

  Subclass of 'admissible' models

Logical Truths & Synthetic Truths

- If *logical truths* are necessarily true in virtue of logical reasoning alone, and logical reasoning is valid independent of subject matter, then having so wide a range of models (structures) that only logical truths are satisfied in all of them is a reasonable strategy.
- If there are *synthetic truths* that are necessarily true in virtue of the meanings of non-logical vocabulary (in addition to logical vocabulary), then *admissible* models (structures) being a proper subset of all logically permitted ones is also a reasonable strategy.
- Characterization of the class of admissible models need not be limited to the object language's expressive resources. (One can't, for example, state with first-order quantification over times that infinitely many times exist.
- Don't expect proof theory of natural languages to be complete.

What About Contingent Truths?

- Construal of admissible models should account for them.
- Is there a sense in which contingent truths can be explained?
  - Some may be explained as consequences of other contingent truths by the language's model theory (thus also implicitly of synthetic and logical truths).
- Analogous to classical mechanics and truths about the dynamics of material bodies.

How Absolute Is Truth?

- Logical truths seem absolute.
- Analytic truths are arguably absolute.
- Some contingent truths are not very controversial, and may be absolute.
  - We are all in Tübingen on August 22, 2014.
- What about
  - The earth revolves around the sun.
  - The sun revolves around the earth.
- What about the answer to
  - Is light a wave or a particle?

- Even one who wants T-sentences should recognize that they only constitute a test that other things explain which contingent sentences are true.
- Don’t forget Tarski’s argument (from weak assumptions) that no consistent theory of truth is possible for a language that contains its own truth predicate.

Thanks for listening.

Let’s talk!