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Cognitive Science

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Conceptual Spaces as a Basis for Semantic Modelling

Two parts

- **Critical:**
- Classical model theory does not work as a foundation for semantics of natural language
- **Constructive:**
- Conceptual spaces do
- Main target of criticism: Montague style semantics, e.g. as expressed in Lewis's *General semantics*

Model theory is ontologically promiscuous

- Set theory provides too many entities, possible worlds (maximal models) and propositions (set of possible worlds)
- For example, Lewis identifies possible individuals with maximal sets of properties, which in turn are sets of things
- Any attempt to reduce the ontological overflow introduces non-logical features

Model theory is cognitively implausible

- There is no evidence that the brain represents possible worlds or embedded functions

A model-theoretic account of adverbs

- Lewis (1970): ”The intension of an *adverb* is a function from functions from functions from indices to things to functions from indices to truth-values to functions from functions from indices to things to functions from indices to truth-values”

Model theory is cognitively implausible

- There is no evidence that the brain represents possible worlds or embedded functions
- Set theory is too abstract to account for mental representations of meanings
- Almost all computational problems are NP-complete
- Model theory cannot explain how we can *learn* concepts

No constraints on possible meanings

- Predicates are taken as primitives
- The notion of predication in first order languages does not fit with the structure of natural language
- Ben-Yami 2004: ”Ironically, Frege was mislead into a mistaken view of concepts, … by what he time and again warned against: mistaking mere grammatical uniformity for a logical and semantic one. The semantic diversity of predication, acknowledged by logicians from Aristotle’s *Categories* on, disappeared under Frege’s pseudo-homogenous semantic relation of falling under a concept.”

No constraints on possible meanings

- Predicates are taken as primitives
- The notion of predication in first order languages does not fit with the structure of natural language
- All predicates are treated on a par
- Hempel’s and Goodman’s paradoxes in inductive reasoning

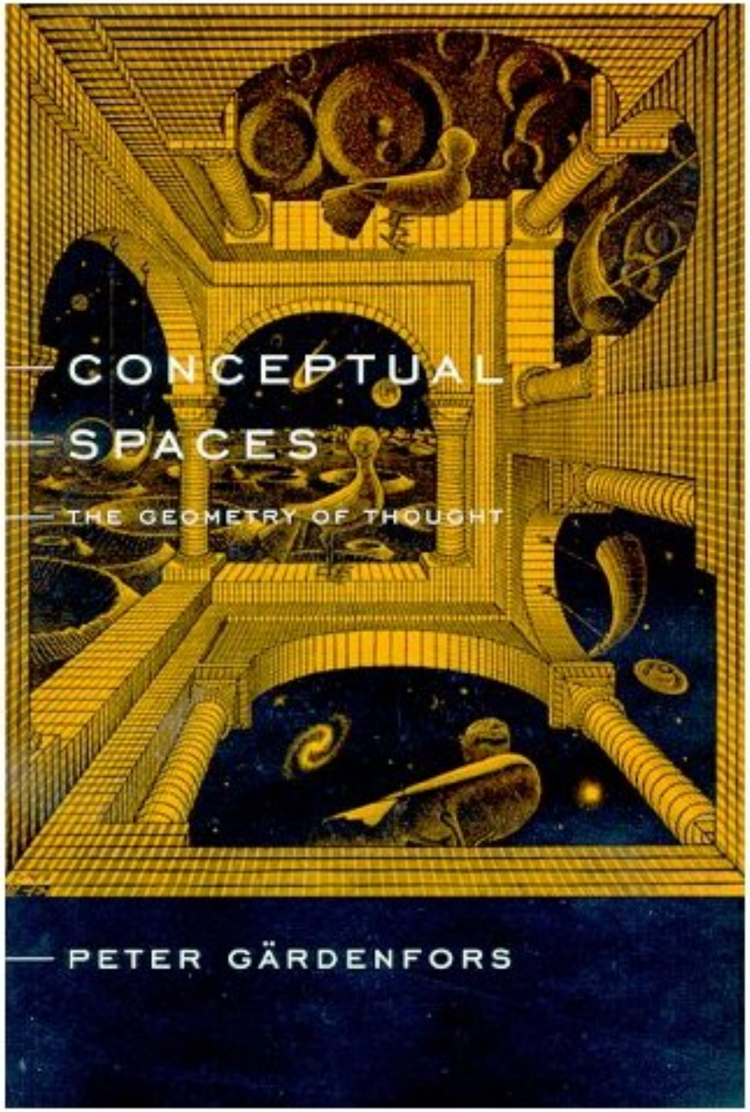
Similarity and metaphor

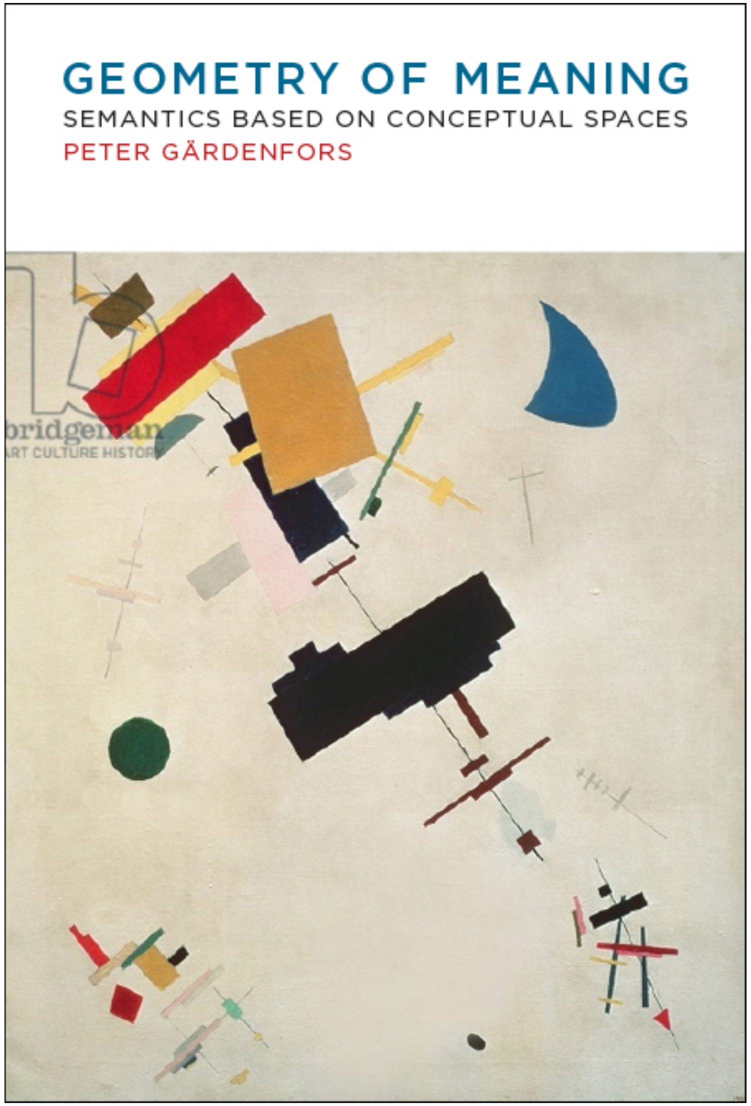
- Similarity relations are ubiquitous in the semantics of natural language
- Metaphors depend on similarities
- Cannot be handled in a natural way within classical model theory

Summary of the critical part

Set theory does not provide the right ontology for modelling the semantics of natural language

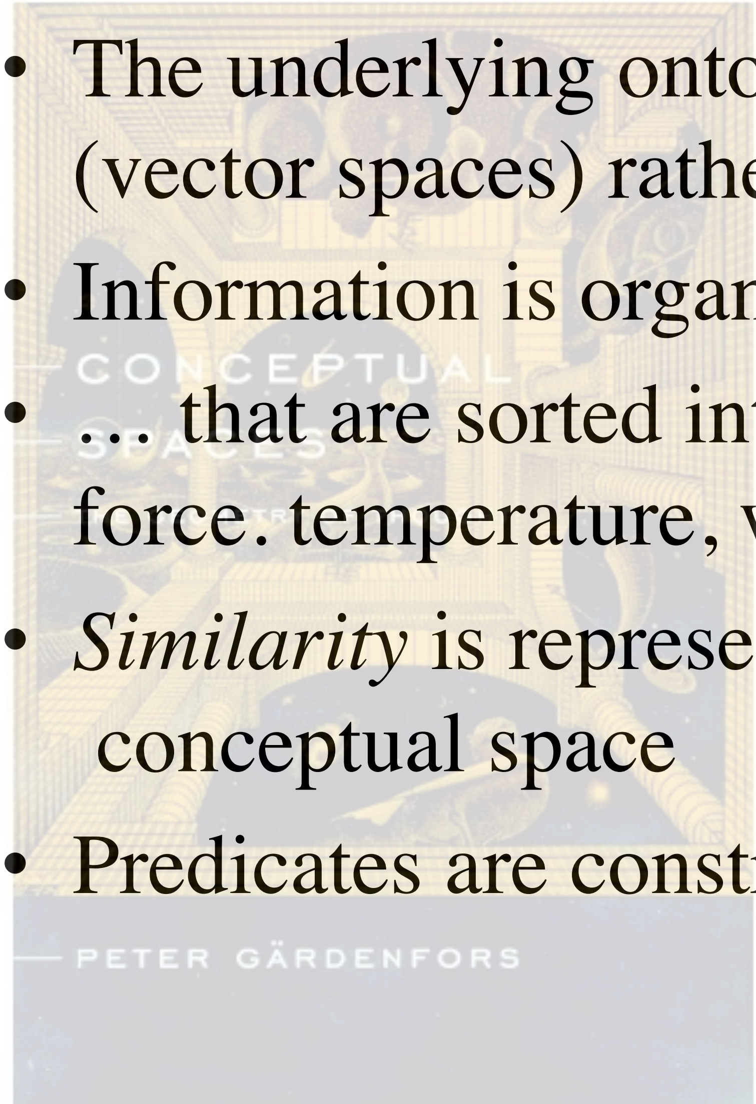
Conceptual spaces

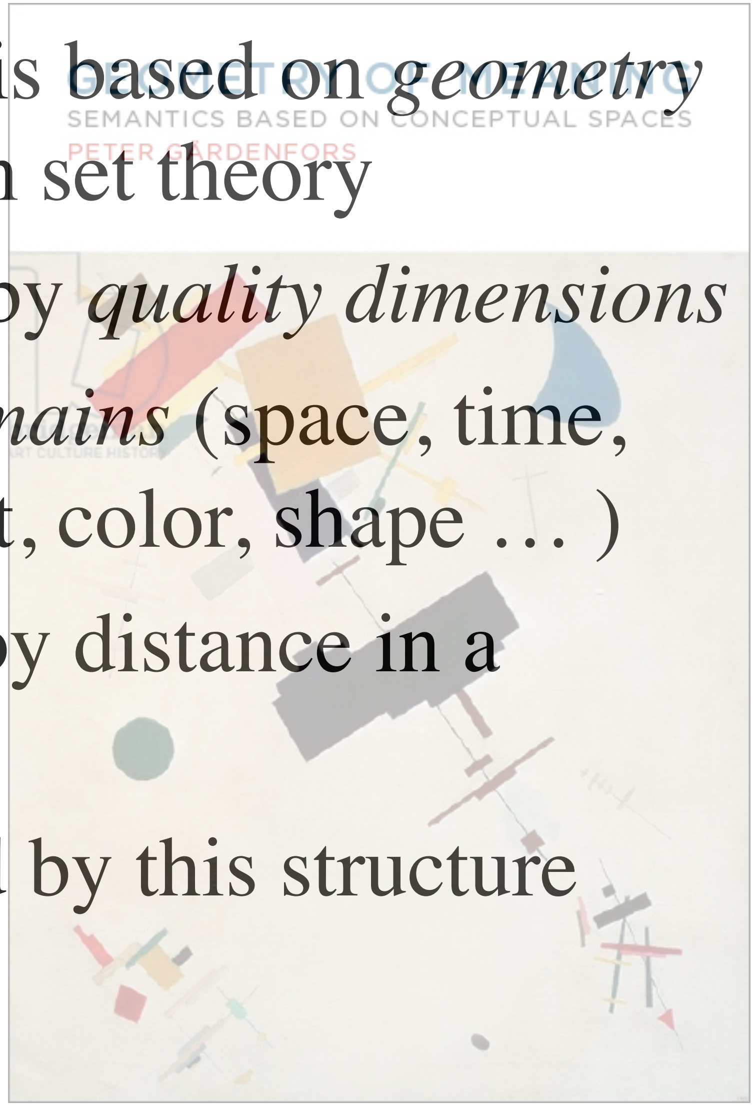




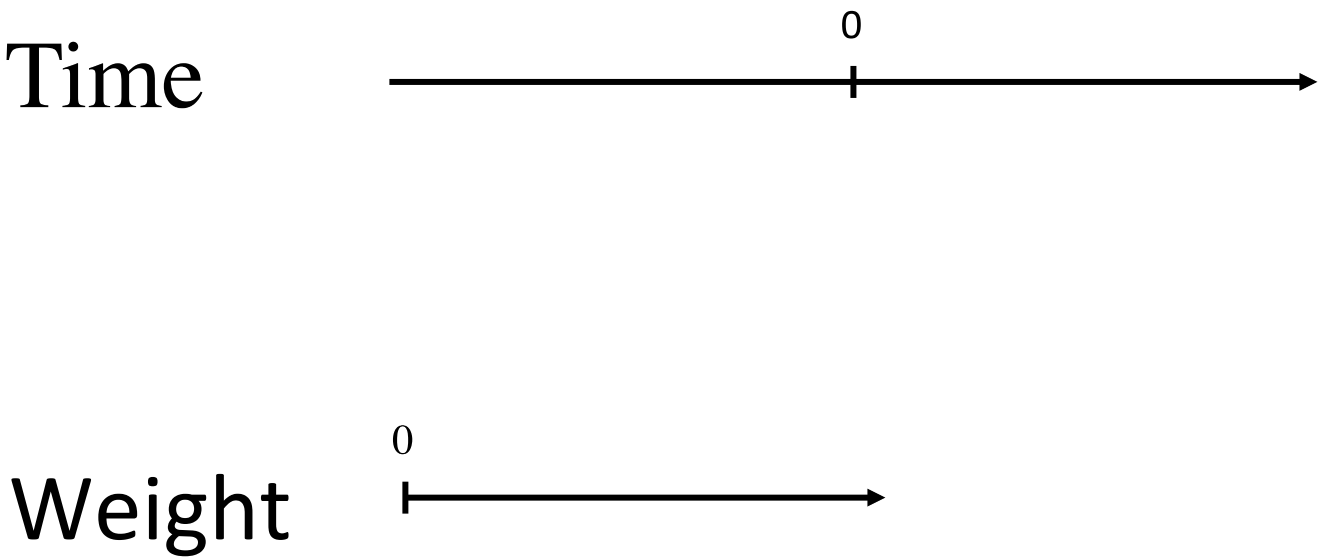
Conceptual spaces

- The underlying ontology is based on *geometry* (vector spaces) rather than set theory
- Information is organized by *quality dimensions*
- … that are sorted into *domains* (space, time, force. temperature, weight, color, shape …)
- *Similarity* is represented by distance in a conceptual space
- Predicates are constrained by this structure

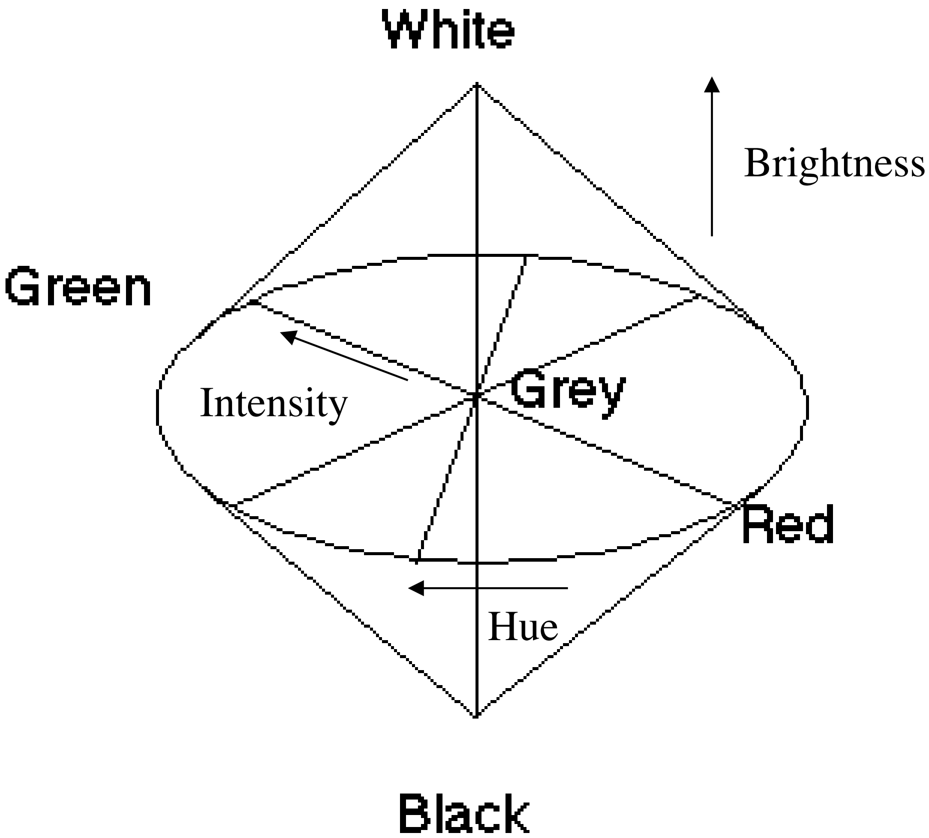




Two linear quality dimensions



The color spindle



The conceptual space of Newtonian mechanics

- *Space*: 3-dimensional Euclidean R^3
- *Time*: 1-dimensional R
- *Mass*: 1-dimensional R^+
- *Force*: 3-dimensional Euclidean vector space R^3

F = a·m defines a *hypersurface* in this 8-dimensional space

The conceptual approach provides a more natural account of scientific theories than structuralism that is based on set theory (Gärdenfors and Zenker 2011, 2013)

Properties vs. concepts

- *Properties*: A convex region in a single domain

Properties vs. concepts

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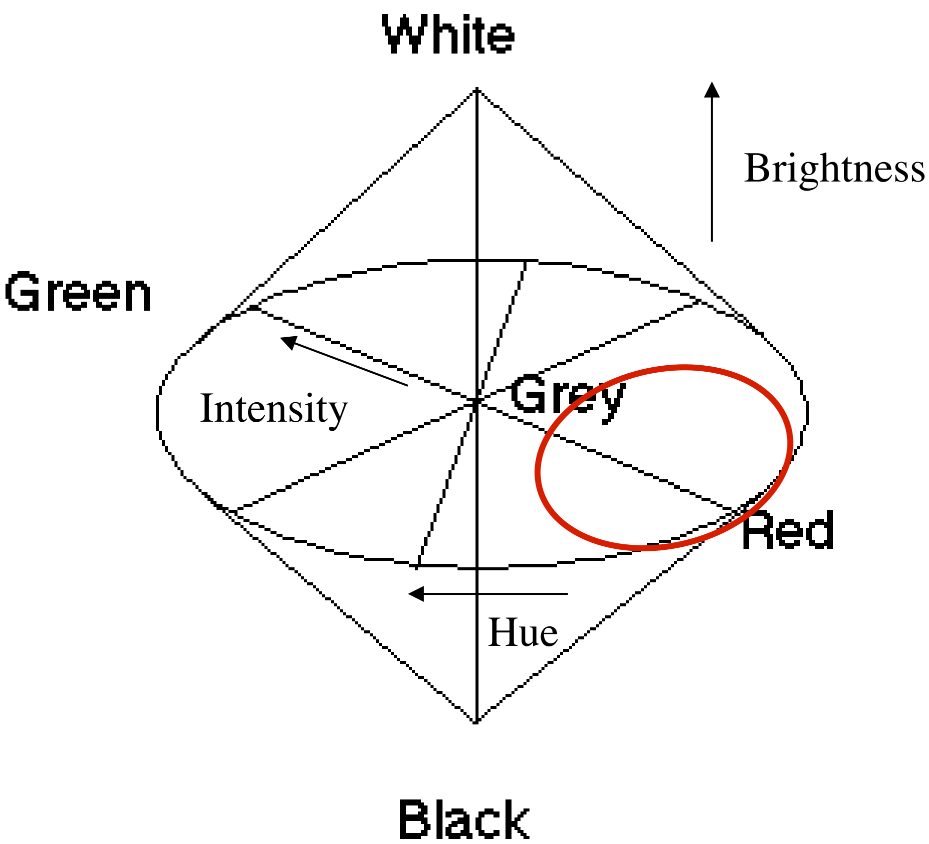
Properties vs. concepts

- *Properties*: A convex region in a single domain
- *Concepts*: A set of convex regions in a number of domains

The color spindle



Properties vs. concepts

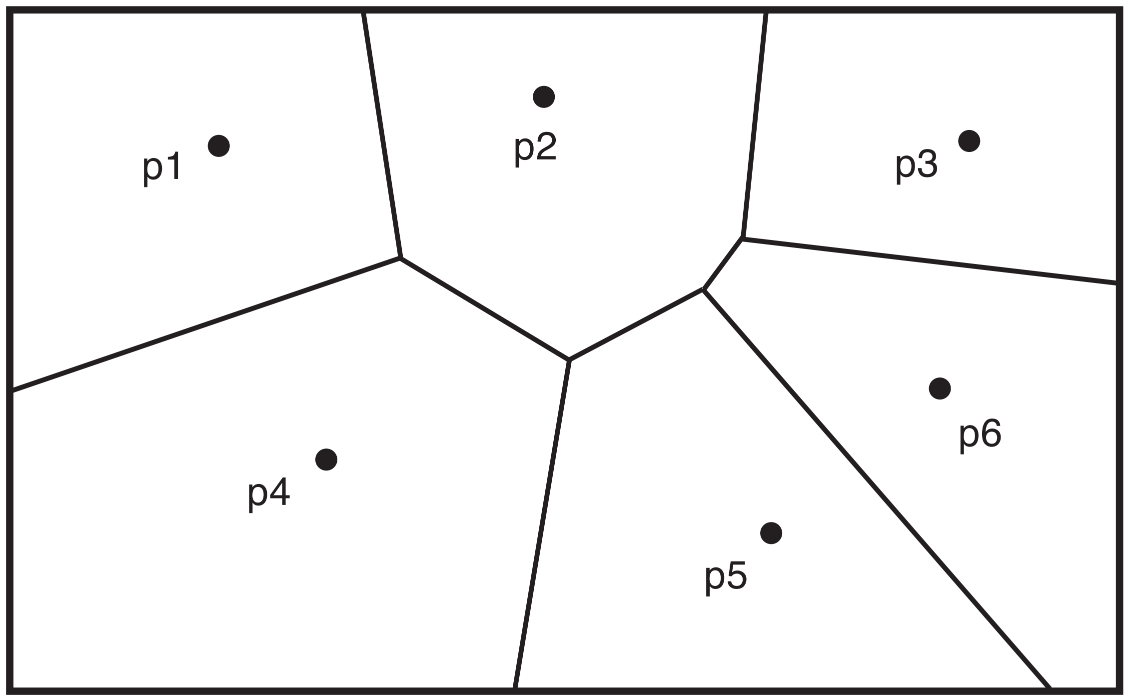
- *Properties*: A convex region in a single domain
- *Concepts*: A set of convex regions in a number of domains; together with (1) prominence values of the domains and (2) information about how the regions in different domains are correlated
- *Adjectives* typically refer to properties, *nouns* to concepts

Models in conceptual spaces

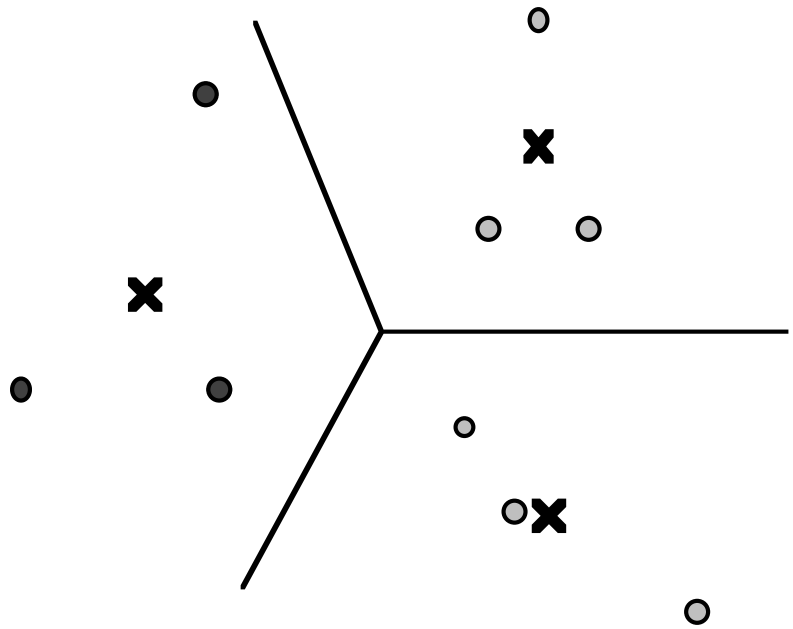
- A conceptual space is a product of domains
- Individuals are represented by points in the space
- Content words (nouns, adjectives, verbs, prepositions, etc) are represented by geometric constructs over the space
- More details in *Geometry of Meaning*
- Neo-Kantian epistemology

Categorization in conceptual spaces

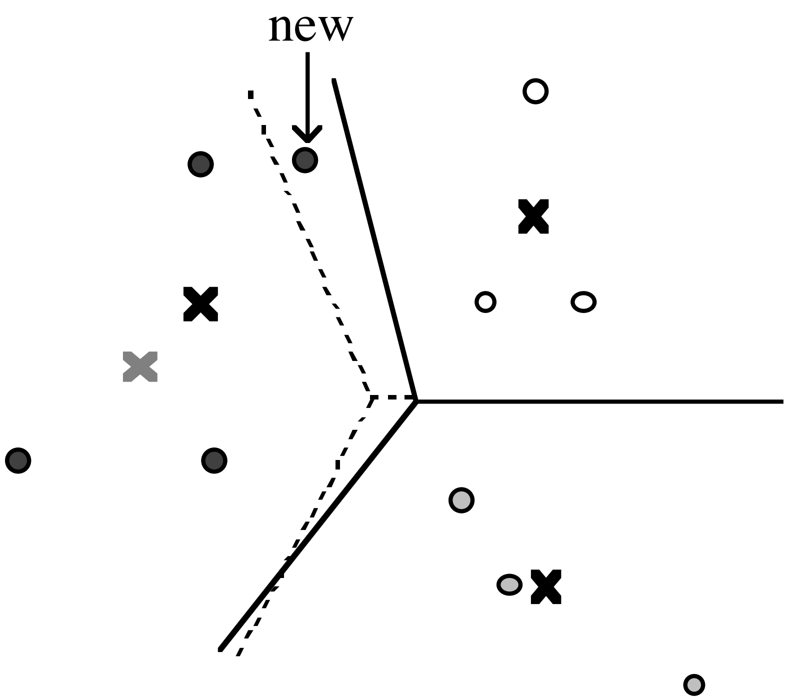
Voronoi tessellations around prototype objects divides conceptual spaces into categories based on the nearest neighbour rule, i.e. each object is associated with the prototype closest to it.



Learning from few examples

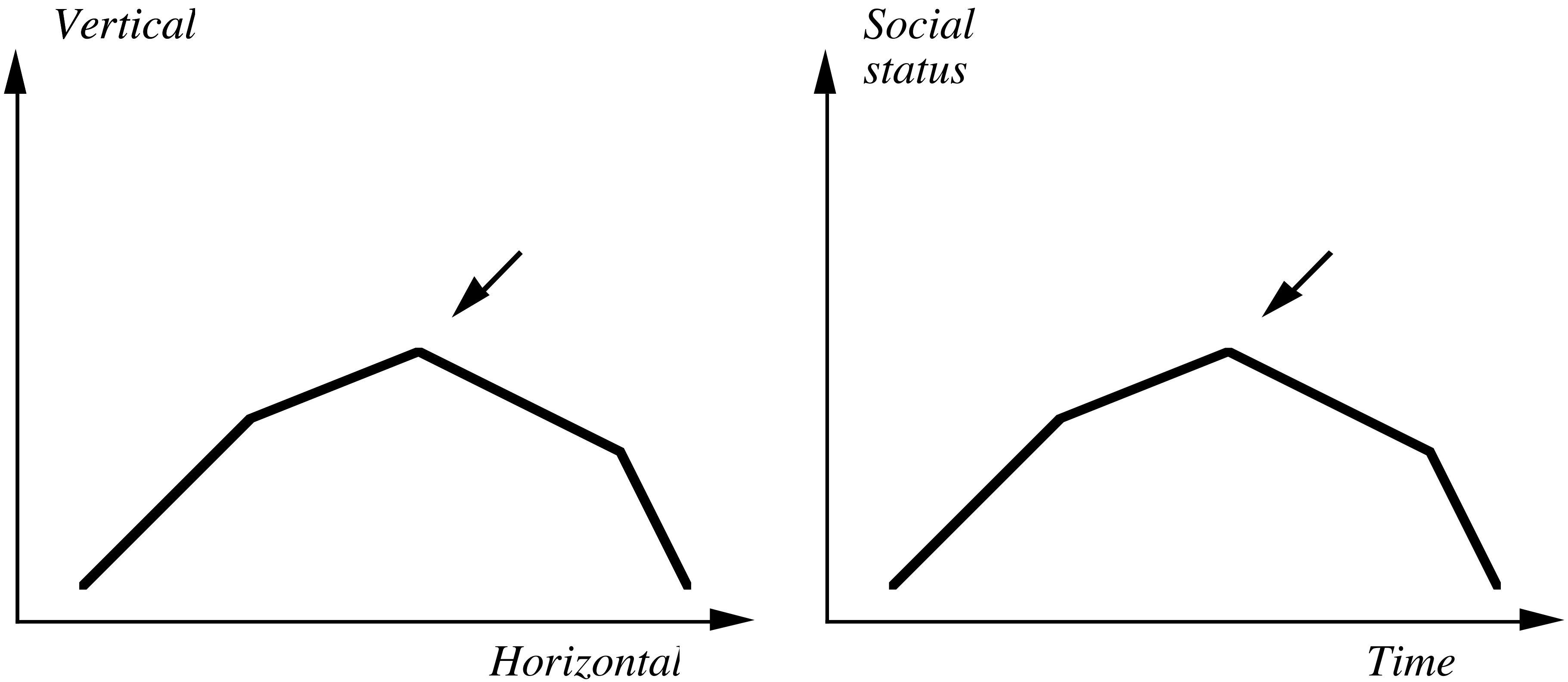


Learning from few examples



The mechanism of metaphor

"The *peak* of a career"



Computational aspects

- *Logical:* Turing machine computations. Recursive operations. Search algorithms.
- *Conceptual spaces:* Vector representations. Calculations involve matrix multiplications (coordinate transformations), inner products, Voronoi tessellations, etc

The structure of the spaces reduces computational complexity

- Classical AI problem: stack of blocks
- Is x above y?
- on(a,b), on(b,c), on(c,d)
- ”Above” as transitive closure of ”on”
- Logical representation results in complexity of order n^2
- Using the vertical dimension as a search constraint gives complexity of order n

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