Classics on Questions II:
Groenendijk & Stokhof

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Questions and Inquisitive Semantics

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1 Background

- By 1997, the study of questions had become a vast area of research; the paper is written with lots of hindsight.
- Groenendijk and Stokhof (GS) did major work in the area in their dissertation Groenendijk and Stokhof (1984) and other papers published around that time.
- Mostly interested in the *logic* of questions (and information exchange) and pragmatic phenomena. Less good a place to look for detailed semantic analyses of particular expressions.
- One of the main goals: to explore relations between questions, as well as between questions and potential answers.

2 Main idea

- A question $?\phi$ denotes a partition $[?\phi]$ of the logical space of possible worlds.
- The *extension* of the question at world w is that member of $[?\phi]$ which is true at w.

- The first point is Hamblin's. Two elements:
  - Existence: The denotations of questions are always non-empty.
    (Recall Hamblin's point bout "residual" answers in case of presupposition failure.)
  - Uniqueness: At any given world, no more than one of the possible answers is true.
    (Potential problem: Mention-some readings. Solution: There are cases in which the complete answer is not what the questioner wants.)

2.1 Question denotations (propositional case)

Let ϕ be a (declarative) sentence and W a set of possible worlds. Along with GS's set notation, I give corresponding functions (for which we need a logic with variables over worlds).

- Extension of ϕ: Truth value.

$$[\phi]_{M,w} \in \{0, 1\}$$

- Intension of ϕ: Proposition.

$$[\phi]_M = \{w \in M | [\phi]_{M,w} = 1\}$$
$$\lambda w.[\phi]_{M,w}$$

- "Intensional interpretation of interrogatives":

$$[?\phi]_{M,w} = \{w' \in M | [\phi]_{M,w'} = [\phi]_{M,w}\}$$
$$\lambda w'.[[\phi]_{M,w'} = [\phi]_{M,w}]$$

- Remarks:
  - Notice that $[?\phi]_{M,w}$ is the *extension* of the question.
  - ⇒ The extension of the question is not an extension (i.e., not a truth value) but an intension (i.e., a proposition). That's what's "intensional" about it.
  - What's the *intension* of the question?

$$[?\phi]_M = \{\langle w, w'\rangle | [\phi]_{M,w} = [\psi]_{M,w'}\}$$
$$\lambda w \lambda w'[[\phi]_{M,w} = [\phi]_{M,w'}]$$

  - Type: $\langle s, \langle s, t\rangle\rangle$ — a *relation between possible worlds*.
    More specifically, an *equivalence relation* (reflexive, transitive, symmetric).
  - Below, we will see a serious problem with this system; for now, let's move along.

2.2 Logical relations

- Entailment: $[?\phi] \models [?\psi] \iff \forall M \forall w \in M.[?\phi]_{M,w} \subseteq [?\psi]_{M,w}$
  "$?\phi$ entails $?\psi$ iff (necessarily) every complete answer to $?\phi$ entails some answer to $?\psi$."
- Equivalence: $[?\phi] \equiv [?\psi] \iff \forall M \forall w \in M.[?\phi]_{M,w} = [?\psi]_{M,w}$
  "$?\phi$ and $?\psi$ are equivalent iff they denote the same partitions."
- Answerhood: $\phi \models ?\psi \iff \forall M \exists w \in M[[\phi]_M \subseteq [?\psi]_{M,w}]$
  "ϕ is an answer to $?\psi$ iff ϕ necessarily entails some complete answer to $?\psi$."
- Remarks:
  - These are relations between semantic objects, rather than the sentences that refer to them. In the latter sense, things may be a bit more complicated if, for instance, not all questions can be expressed.
  - I'm not sure about their use of the symbol '$\models$' for answerhood.

2.3 Predicate logic

GS gloss over the intricacies involved in WH-questions, assuming (for simplicity) that the '?' operator will bind all free variables in the sentence it combines with.

- Preliminaries:
  - Expressions of the form '$?x_1, \ldots, x_n\phi$' ($n \geq 0$)
    (Note: You will sometimes see '$?\vec{x}\phi$' as an abbreviation.)
  - Worlds are first-order models, assigning extensions to constants;
    g is an assignment function taking variables to individuals.
- Interpretation:

$$\langle ?x_1, \ldots, x_n\phi\rangle_{M,w,g} = \{\langle g'(x_1), \ldots, g'(x_n)\rangle | [\phi]_{M,w,g'} = 1\}$$
$$\text{where } g'(x) = g(x) \text{ for all } x \neq x_1, \ldots, x_n$$

(the set of n-tuples of individuals which satisfy ϕ in w)

$$[?\vec{x}\phi]_{M,w,g} = \{w' \in M | \langle ?\vec{x}\phi\rangle_{M,w,g} = \langle ?\vec{x}\phi\rangle_{M,w',g}\}$$

(the set of worlds in which the same n-tuples of individuals satisfy ϕ as in w)

2.4 A simple example

(1) a. Who danced?
b. $?x.dance'(x)$

Consider some model M. For simplicity, assume that the domain D of individuals is constant across all worlds. Also assume that there are "enough" worlds to represent all possible assignments of extenstions to predicates like $dance'$. Let $g(x) = a$ for all x. I will drop the subscript 'M' throughout.

$$M = \{u, v, w, \ldots\}$$
$$D = \{a, b, c\}$$
$$dance' = \begin{bmatrix} u \mapsto & \{a, b\} \\ v \mapsto & \{a, b, c\} \\ w \mapsto & \{a, b\} \\ \ldots & \end{bmatrix}$$

$$\begin{aligned} \langle ?x.dance'(x)\rangle_{u,g} &= \{g'(x) | [dance'(x)]_{u,g'} = 1\} \\ &= \{a, b\} \end{aligned}$$

$$\begin{aligned} [?x.dance'(x)]_{u,g} &= \{\omega \in M | \langle ?x.dance'(x)\rangle_{\omega,g} = \langle ?x.dance'(x)\rangle_{u,g}\} \\ &= \{\omega \in M | \langle ?x.dance'(x)\rangle_{\omega,g} = \{a, b\}\} \\ &= \{u, w, \ldots\} \end{aligned}$$

Similarly for the other worlds. Thus we get a partition like this (showing where our three worlds are located):

$\emptyset$	
$\{a\}$	
$\ldots$	
$\{a,b\}$	u,w
$\ldots$	
$\{a,b,c\}$	v

Cf. also GS, page 1092, Fig. 2.

Thus given GS's assumptions, (1) has the same extension at *u* and *w*, and a different one at *v*. (Recall that its *intension*, the partition, is the same at all worlds.)

2.5 A (slightly) more complex example

(2) a. Which student(s) danced?
b. $?x.student'(x) \wedge danced'(x)$

Let the model be as above, with the following addition:

$$student' = \begin{bmatrix} u \mapsto \{a,b\} \\ v \mapsto \{b,c\} \\ w \mapsto \{a,b,c\} \\ \ldots \end{bmatrix}$$

Now the question denotation at each world depends on which individuals are both students and dancers.

$$\begin{aligned}\langle ?x.student'(x) \wedge dance'(x)\rangle_{M,u,g} &= \{g'(x) | [student'(x) \wedge dance'(x)]_{M,u,g'} = 1\} \\ &= \{a,b\}\end{aligned}$$

$$\begin{aligned}[?x.student'(x) \wedge dance'(x)]_{M,u,g} &= \{\omega \in M | \langle ?x.student'(x) \wedge dance'(x)\rangle_{M,\omega,g} = \{a,b\}\} \\ &= \{u,w,\ldots\}\end{aligned}$$

And so on for the other worlds.

2.6 A problem

- Thus it is predicted that (2) has the same extension at *u* and *w*.
- But does the question really *mean* the same at both of these worlds?
  (I.e., does it have the same possible answers?)
- *No*.
  - Consider the following two answers:

    (3) a. *a* and *b* dance; *c* and *d* don't.
    b. *a*, *b* and *c* dance; *d* doesn't.
  - At *u*, (3a,b) are (over-specifications of) the *same* answer.
  - At *w*, (3a,b) are *different* answers.
  - Intuitively, (2) denotes *different partitions*: 4 cells at *u*; 8 cells at *w*.
  - Compare (2) with the following:

    (4) Who is a student who danced?

- What should we do about this?
  - say that the *extension* of a question is a partition, and that its *intension* is a function from possible worlds to partitions (i.e., type $\langle s, \langle s, \langle s, t \rangle\rangle\rangle$)?

    But more would be required: E.g., say that (3) is a question *about the students* (at the world of evaluation), as opposed to (5):

    (5) Who is a student and danced?
  - say that the extension of a question is not a proposition, but a family of propositions with algebraic structure; at each world, one of them is "preferred"; but the family is the same?
  - assume for now that the denotations of the relevant predicates are constant across possible worlds?
- A somewhat related problem is mentioned in Section 4.5.5, p. 1096.

2.7 Logical relations

- Entailment (4.17, p. 1093): A sequence of questions entails $?\phi_1, \ldots, ?\phi_n$ entails a question $?\psi$ iff every consistent collection of answers to each of $?\phi_1, \ldots, ?\phi_n$ jointly entails an answer to $?\psi$.

  (Some consequences of this definition are listed in Fact 4.18, p. 1093.)
- Answerhood (4.19, p. 1094): ϕ is an answer to $?\psi$ iff it is contained in one of the cells in $?\phi$'s partition.

  (Some consequences in Fact 4.20.)

2.7.1 Comparative answerhood

- Informativeness: ϕ gives a partial true answer to $?\psi$ at w iff it overlaps with (i.e., differentiates between worlds within) w's cell in the partition.

  (Note: The term is potentially confusing. A "partial true answer" is not necessarily true at w; it only needs to overlap with the true anser!)
- ϕ is a more informative answer to $?\psi$ than ϕ' iff ϕ' overlaps with all the cells in the partition that ϕ overlaps with (and possibly more).

  (Note: The order on possible answers thus defined is a pre-order, not a partial order; hence it should better be "at least as informative as," not "more informative than.")
- Comparing answers: If ϕ and ϕ' are partial true answers[1] to ψ at w, then ϕ is a *better* answer than ϕ' iff
  - ϕ is more informative than ϕ'; or
  - else, ϕ is entailed by ϕ'.

  (Note: This definition only works with respect to some particular world; it doesn't quite give a general definition of overall "goodness.")

[1] They write "true partial," but that's probably an error.

3 Some other things

3.1 De dicto vs. de re

(6) Which student(s) passed the exam?

- Karttunen:

  (7) $\lambda p[\exists x[student'(x) \wedge {}^{\vee}p \wedge p = {}^{\wedge}pass'_{*}({}^{\vee}x)]]$

- As GS point out, the propositions in this set (thinking of it as a set) are those which assert *of some student* (in the world of evaluation) that that individual passed.

- Ech of these p may contain worlds at which x is not a student (but passed the exam). An alternative reading:

  (8) $\lambda p[{}^{\vee}p \wedge \exists x[p = {}^{\wedge}(student'(x) \wedge pass'(x))]]$

- This also has potential implications for questions like (9) in our world:

  (9) Which unicorn ate the beets?

  for which we might prefer this:

  (10) $\lambda p[{}^{\vee}p \wedge p = {}^{\wedge}\exists x[uni'(x) \wedge atb'(x)]]$

- Karttunen's system doesn't get these because wh-phrases have to be quantified into (proto-) questions and can't come in before the proto-question is formed.

3.2 Conjunction

Recall that neither of (11b,c) is equivalent to (11a).

(11) a. Will John be there? And will Mary be there?
b. Will John and Mary be there?
c. Will John or Mary be there?

GS note that the meaning of (11a) seems to be properly captured by the pairwise intersection of their respective paritItions. They don't give a linguistic account of how this comes about, though.

3.3 Glimpses of further developments

Groenendijk (1999) applies his partition semantics to longer discourses in which "issues" are raised and resolved by the interlocutors.

- Question denotation at world w:

$$[\![?\vec{x}\psi]\!]_{w,g} = \{v \in W | \forall \vec{e} \in D^n : [\![\phi]\!]_{v,g[\vec{x}/\vec{e}]} = [\![\phi]\!]_{w,g[\vec{x}/\vec{e}]}\}$$

- Context C: a symmetric and transitive relation on the set W of possible worlds.

  (One can (and maybe should) also require that it be either reflexive or euclidean, but G doesn't do so. But he does define '$w \in C$' as '$\langle w, w\rangle \in C$'.)

- Absurd context: $\emptyset$
- Indifferent context C: one for which $\langle w, v\rangle \in C$ for all $w, v \in C$.
- Context change potentials (writing 'ϕ!' and 'ϕ?' for ccp's of assertions and questions, respectively):

$$C[\phi!] = \{\langle w, v\rangle \in C | [\![\phi!]\!]_w = [\![\phi!]\!]_v = 1\}$$
$$C[\phi?] = \{\langle w, v\rangle \in C | [\![\phi?]\!]_w = [\![\phi?]\!]_v\}$$
$$\text{For } \tau = \phi_1; \ldots; \phi_n, C[\tau] = C[\phi_1] \ldots [\phi_n]$$

  Note: $C[\phi] \subseteq C$ for all C, ϕ.

- Entailment:

$$\tau \models \phi \iff \forall C : C[\tau] = C[\tau][\phi]$$

  If ϕ is an assertion: Once τ is processed, ϕ does not add new information. If ϕ is a question: It does not raise an issue that has not yet been addressed.

- Licensing: τ licences ϕ iff for all C, w, v:

$$\langle w, v\rangle \in C[\tau] \wedge w \notin C[\tau][\phi] \Rightarrow v \notin C[\tau][\phi]$$

  That is, ϕ is licensed after τ if it removes an entire cell from the partition, without "cutting across" any cells.

  (Notice that unless an issue is raised in $C[\tau]$, only $\emptyset$ and propositions non-informative propositions would be licensed. I guess the definition is only intended for answers to questions. But it is a very strong condition there, too.)

- Pertinence: ϕ is pertinent after τ iff

  a. ϕ is consistent with τ
  b. ϕ is not entailed by τ
  c. ϕ is licensed after τ

- And so on. G goes on to define a whole list of other notions. Their use in explicating linguistic phenomena has yet to be established, but it's a promising start.

References

Groenendijk, J. 1999. The logic of interrogation. In Matthews, T. and D. Strolovitch, editors, *Proceedings of SALT IX*. CLC Publications.

Groenendijk, J. and M. Stokhof. 1984. *Studies in the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam.