J Philos Logic (2009) 38:1–53
DOI 10.1007/s10992-008-9088-0

Conditionals Right and Left: Probabilities for the Whole Family

Stefan Kaufmann

Received: 10 January 2006 / Accepted: 21 September 2006 / Published online: 8 August 2008

Abstract The fact that the standard probabilistic calculus does not define probabilities for sentences with embedded conditionals is a fundamental problem for the probabilistic theory of conditionals. Several authors have explored ways to assign probabilities to such sentences, but those proposals have come under criticism for making counterintuitive predictions. This paper examines the source of the problematic predictions and proposes an amendment which corrects them in a principled way. The account brings intuitions about counterfactual conditionals to bear on the interpretation of indicatives and relies on the notion of causal (in)dependence.

Keywords Standard probabilistic calculus · Embedded conditionals · Probabilistic theory of conditionals · Causal independence

1 Introduction

Theories of reasoning, decision-making and communication under uncertainty invariably must deal with degrees of belief or certitude. Often, the methodological tool of choice in modeling such degrees is the Bayesian probabilistic calculus. The probability assigned to an atomic sentence is the probability that it is true. This interpretation naturally extends to Boolean compounds of sentences: Their truth-functional interpretation determines how their probabilities depend on the probabilities of their constituents and the way they are combined.

S. Kaufmann (✉)
Department of Linguistics, Northwestern University,
2016 Sheridan Road, Evanston, IL 60208-4090, USA
e-mail: kaufmann@northwestern.edu

This connection between truth and probability breaks down in the case of conditional (*'if-then'*) sentences. A widely shared opinion holds that the probability of a sentence *'if A, then C'* is the conditional probability of C, given A (henceforth "The Thesis"; see Ramsey [34]; Jeffrey [19]; Adams [1, 2, 4]; Stalnaker [39]; McGee [30]; Stalnaker and Jeffrey [40], and many others). To what extent the two probabilities do indeed coincide is an empirical question, on which there has been some controversy ([11, 21, 23, 31, 33], also the discussion below). But granting that they coincide at least for a large and significant class of cases, the task of building this equivalence into a logical theory proves to be formidable. A conditional probability cannot be interpreted as the probability that a proposition is true [14, 26, 28], thus an answer to the question of how the probability of a conditional depends on the probabilities of its constituents cannot be given in terms of truth conditions in the familiar way.

Still, intuitions about conditionals must be accounted for.[1] In view of the above difficulties, some authors seek to build such an account on conditional probabilities alone, either reducing the truth values of conditionals to rudiments of little semantic or pragmatic use ([16–18]; see Appiah [5]; Ellis [10], for criticism), or abandoning the idea that conditionals have truth values altogether [2, 4]. But such moves create new problems of their own. The probabilistic calculus, in conjunction with The Thesis, does not define probabilities for sentences containing embedded conditionals. Such sentences exist, however: All the examples in (1) are well-formed, and speakers have fairly clear intuitions about the circumstances under which they have high or low probabilities.

(1) a. If this match is wet, it won't light if you strike it.
b. If Martha is in the kitchen, we'll have dinner soon, and if Marv is in the garage, the car will be fixed tonight.
c. If this switch will fail if it is submerged in water, it will be discarded.
d. If this vase will crack if it is dropped on wood, it will shatter if it is dropped on marble.

One way to avoid this issue is to deny that complex conditionals make sense, or to claim that speakers treat them as equivalent to simpler sentences [6, 12, 18, 42]. Alternatively, some authors have pushed the notion of what kinds of denotations conditionals can have, developing ingenious ways of obtaining probabilities for more complex sentences [20, 30, 40, 41]. However,

[1] To be clear about what I take to be the goal of this endeavor: I do not believe that conditionals are, or denote, a logical relation between sentences. Such a view is attested and has rightfully been blamed on the widespread habit of referring to the sentential connectives '$\supset$' and '-3' as material and strict "implication," respectively. The confusion can be traced back to Russell [35] and was discussed by Sanford [36] and Kyburg et al. [24], among others. I do believe, however, that the question of how the probabilities of complex sentences depend on the probabilities of their constituents (and possibly other factors) is of genuine logical interest, and it is this question that I seek to answer in this paper.

the probabilities that are predicted under this latter approach have come under criticism for being counterintuitive in particular cases [8, 25, 29].

Although these criticisms of the latter approach are justified with regard to the particular proposals they were directed at, I nevertheless believe that the approach itself holds the key to the proper treatment of complex conditionals. This paper offers a new and refined version of the approach which corrects the problems identified by its critics. The proposal is an extension of Kaufmann's [22] treatment of conditionals with conditional consequents. That proposal does not extend to conjunctions of conditionals and conditionals with conditional antecedents. To cover these cases, I will build on the model theory introduced by van Fraassen [41], discussed in detail in Section 2 below. The use of this formal apparatus for the assignment of truth values to arbitrarily complex conditionals was previously suggested by Stalnaker and Jeffrey [40]. However, much is left implicit in their discussion, and it is not immediately obvious to the casual reader what the predicted truth values actually are. Such an explicit specification is helpful for the purposes of this paper, and I will spell it out in Section 3.

Section 4 introduces an amendment to the account which is designed to correct the counterintuitive predictions, and Section 5 illustrates the effect of the amendment with a number of examples. Two recurring themes in this discussion will be *counterfactuals* and *causal independence*. Following Kaufmann [22], I will argue that the former are a valuable source of intuitions about the assignment of truth values to indicative conditionals. Causal relations will figure prominently in the formal definitions I am going to propose. Both of these elements were already present in Kaufmann's [22] proposal for right-nested conditionals. In extending the general idea to a wider range of data, the present paper adds arguments in favor of that approach and takes a further step towards a comprehensive, technically sound and intuitively plausible probabilistic semantic theory of indicative and counterfactual conditionals.

2 Theoretical Background

The product space construction by van Fraassen [41] was the first demonstration that it is possible to give a principled probabilistic interpretation to a language which includes freely compounded and embedded conditionals. I will introduce the idea in this section in enough detail to sustain the discussion in the rest of the paper. The reader is referred to van Fraassen's paper for further details and proofs.

2.1 Languages

Based on a set $\mathcal{A}$ of atomic sentences, I start by defining two successively complex languages. I use Greek letters φ, ψ etc. as variables ranging over all sentences, atomic and complex. I write '$\varphi\psi$' instead of '$\varphi \wedge \psi$' and '$\overline{\varphi}$' for '$\neg\varphi$'.

The connective '$\rightarrow$' is not the material conditional, but rather the probabilistic connective whose interpretation the remainder of this paper is devoted to.

Definition 1 (Languages) Given a set $\mathcal{A}$ of atomic sentences, we define two languages as follows:

a. $\mathcal{L}^0_{\mathcal{A}}$ is the smallest set containing $\mathcal{A}$ and such that for all $\varphi, \psi \in \mathcal{L}^0_{\mathcal{A}}$, $\overline{\varphi}$, $\varphi\psi \in \mathcal{L}^0_{\mathcal{A}}$.
b. $\mathcal{L}_{\mathcal{A}}$ is the smallest set containing $\mathcal{A}$ and such that for all $\varphi, \psi \in \mathcal{L}_{\mathcal{A}}$, $\overline{\varphi}$, $\varphi\psi$, $\varphi \rightarrow \psi \in \mathcal{L}_{\mathcal{A}}$.

Thus $\mathcal{L}^0_{\mathcal{A}}$ is closed only under truth-functional connectives (others, such as disjunction or the material conditional, may be defined in terms of conjunction and negation as usual). $\mathcal{L}_{\mathcal{A}}$ is not subject to this restriction, featuring free compounds and embeddings of conditionals. In the following, I will occasionally use Roman letters $A, B, \ldots$ as variables restricted to range over sentences in $\mathcal{L}^0_{\mathcal{A}}$.

2.2 Probability Models

The languages $\mathcal{L}^0_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ are interpreted with respect to structures which determine their truth values and probabilities. For clarity of exposition, in this section I will only discuss the interpretation of $\mathcal{L}^0_{\mathcal{A}}$ in such models. This will eventually be extended to the full language $\mathcal{L}_{\mathcal{A}}$, but only after some more definitions, which are introduced in the next subsection.

A probability model has two ingredients, a probability space and a truth assignment.

Definition 2 (Probability space) A *probability space* is a triple $\langle \Omega, \mathcal{F}, \Pr \rangle$, where Ω is a non-empty countable set, $\mathcal{F}$ is the powerset of Ω, and Pr is a probability measure on $\mathcal{F}$.[2]

I will refer to the elements of Ω as "possible worlds." For Pr, as well as the other probability functions defined below, I will use the familiar notation '$\Pr(Y|X)$' as shorthand for '$\Pr(Y \cap X)/\Pr(X)$'.[3]

[2] Van Fraassen [41] and Stalnaker and Jeffrey [40] do not include the requirement that $\mathcal{F}$ be the powerset of Ω. In general, any σ-algebra over Ω would do in generating a probability space. (A σ-algebra over Ω is a non-empty set of subsets of Ω that contains Ω and is closed under complement and countable union.) Taking $\mathcal{F}$ to be the powerset of Ω simplifies matters and is harmless here because Ω is countable. Pr is a probability measure on $\mathcal{F}$ if $\Pr(\Omega) = 1$ and for all countable sequences $X_1, \ldots X_n$ of pairwise disjoint sets $X_i \in \mathcal{F}$, $\Pr(\bigcup_{i=1}^n X_i) = \sum_{i=1}^n \Pr(X_i)$.

[3] Thus $\Pr(Y|X)$ is undefined if $\Pr(X) = 0$. This property is common to many probabilistic models, but not that of Stalnaker and Jeffrey [40], which is similar to the present account in other respects. Stalnaker and Jeffrey impose the multiplicative law $\Pr(Y|X)\Pr(X) = \Pr(Y \cap X)$ as an axiom, rather than using Bayes's rule as a definition, allowing for arbitrary values of $\Pr(Y|X)$ when $\Pr(X) = 0$. The difference does not affect the semantic theory developed in this paper.

Definition 3 (Probability model) Let $\langle \Omega, \mathcal{F}, \Pr \rangle$ be a probability space and $V_{\mathcal{A}} : \mathcal{A} \mapsto (\Omega \mapsto \{0, 1\})$ a truth assignment to the atomic sentences in $\mathcal{A}$ at worlds in Ω. A *probability model for* $\mathcal{L}^0_{\mathcal{A}}$ is a quadruple $\langle \Omega, \mathcal{F}, \Pr, \mathrm{V} \rangle$, where V is an assignment of truth values to sentences in $\mathcal{L}^0_{\mathcal{A}}$, subject to the following conditions: For all $\varphi, \psi \in \mathcal{L}^0_{\mathcal{A}}$ and $\omega \in \Omega$,

$$\text{If } \varphi \in \mathcal{A}, \text{ then } \mathrm{V}(\varphi)(\omega) = V_{\mathcal{A}}(\varphi)(\omega)$$
$$\mathrm{V}(\varphi\psi)(\omega) = \mathrm{V}(\varphi)(\omega) \times \mathrm{V}(\psi)(\omega)$$
$$\mathrm{V}(\overline{\varphi})(\omega) = 1 - \mathrm{V}(\varphi)(\omega)$$

The range of V in Definition 3 is $\{0, 1\}$. In the next subsection, I will extend this assignment to the full conditional language $\mathcal{L}_{\mathcal{A}}$. Once this extension has been introduced, the range of V will be the interval $[0, 1]$, although the values of sentences in $\mathcal{L}^0_{\mathcal{A}}$ will still be in $\{0, 1\}$.

The model and the truth assignment jointly determine the probabilities of sentences according to the following rule.

Definition 4 The probability $\mathrm{P}(\varphi)$ of a sentence φ in a model $\langle \Omega, \mathcal{F}, \Pr, \mathrm{V} \rangle$ is the expectation of its truth value:[4]

$$\mathrm{P}(\varphi) =_{df} E[V(\varphi)] = \sum_{x \in range(V(\varphi))} x \times \Pr(\{\omega \in \Omega | V(\varphi)(\omega) = x\})$$

If $\varphi \in \mathcal{L}^0_{\mathcal{A}}$, the range of $V(\varphi)$ is $\{0, 1\}$ and the formula in Definition 4 simplifies to (2):

(2)
$$\mathrm{P}(\varphi) = \Pr(\{\omega \in \Omega | V(\varphi)(\omega) = 1\})$$

This will no longer be the case once the value assignment is extended to $\mathcal{L}_{\mathcal{A}}$. However, even in that case I do assume for simplicity that the range of the assignment function is finite.

[4]The set $\{\omega \in \Omega | V(\varphi)(\omega) = x\}$ is a subset of Ω, hence a member of $\mathcal{F}$ (for recall that $\mathcal{F}$ is the powerset of Ω), so its probability is defined. Technically, it is the event that the random variable $V(\varphi)$ has value x. A common and more concise way of referring to its probability would be as '$\Pr(V(\varphi) = x)$'. The more explicit set notation I use here will facilitate the statement of more complex descriptions of events in some cases below.

2.3 Stalnaker Bernoulli Models

The extension of the above probability models to $\mathcal{L}_{\mathcal{A}}$ proceeds as follows. First, based on the probability space $\langle \Omega, \mathcal{F}, \Pr \rangle$, the idea is to construct a structure $\langle \Omega^*, \mathcal{F}^*, \Pr^* \rangle$ as follows:

Definition 5 (Stalnaker Bernoulli space—van Fraassen [41]) A *Stalnaker Bernoulli space* based on a probability space $\langle \Omega, \mathcal{F}, \Pr \rangle$ is a structure $\langle \Omega^*, \mathcal{F}^*, \Pr^* \rangle$, where

a. Ω^* is the set of all countable sequences of worlds in Ω. For each $\omega^* \in \Omega^*$ and $n \geqslant 1$, $\omega^*[n]$ denotes the n-th world in ω^*.
b. $\mathcal{F}^*$ is the set of all sequences $X_1 \times \ldots \times X_n \times \Omega^*$, for $n \in \mathbb{N}$, $X_i \in \mathcal{F}$.
c. for all $X_1 \times \ldots \times X_n \times \Omega^* \in \mathcal{F}^*$, $\Pr^*(X_1 \times \ldots \times X_n \times \Omega^*) = \prod_{i=1}^{n} \Pr(X_i)$.

I will refer to the members of Ω^* as "sequences" or "world sequences." The intuitive idea is that each such sequence corresponds to the outcome of an infinite series of random choices of worlds from Ω with replacement, where the expectation of each trial is independent of the previous outcomes and given by the probability measure Pr on members of $\mathcal{F}$, which are subsets of Ω (hence van Fraassen's term "Stalnaker Bernoulli model"). Although $\mathcal{F}$ is the powerset of Ω, $\mathcal{F}^*$ is not the powerset of Ω^*. The association of "starred" symbols with the product space and their correspondence to "unstarred" counterparts in the original space will be a notational convention throughout this paper.

What remains after "lopping off" the first world, or in fact any finite initial sequence of worlds, from a sequence $\omega^* \in \Omega^*$ is again a member of Ω^*. I will refer to such "tails" with the notation '$\omega^*(\cdot)$'. Thus $\omega^*(n)$ is the final subsequence beginning with the n-th member of ω^*. This must not be confused with the notation '$\omega^*[\cdot]$' introduced in Definition 5: Notice that $\omega^*(n) \in \Omega^*$ whereas $\omega^*[n] \in \Omega$.

It is well to clarify a couple of terminological issues at this point in order to avoid confusion later on. From the probability-theoretical perspective that is suggested by the name of the model as well as the intuitive motivation given above, the "worlds" $\omega \in \Omega$ are *outcomes* and the sets $X \in \mathcal{F}$ are *events*. At the same time, semanticists working in the logical tradition in which Stalnaker's work on conditionals has been most influential are accustomed to identifying sets of possible worlds with *propositions*. I use the terms "event" and "proposition" in this technical sense, in which they are interchangeable. Furthermore, I use them also to refer to sets of sequences in $\mathcal{F}^*$. Thus for instance, $X \times \overline{X} \times \Omega^*$ is the event that an X-world is drawn first and a $\overline{X}$-world next, i.e., the set of all sequences ω^* such that $\omega^*[1] \in X$ and $\omega^*[2] \notin X$. Similarly, $\Omega \times X \times \Omega^*$ is the event that an X-world is chosen on the second trial, and so on.

Of particular interest here is the event that an X-world is chosen at *some* point in the sequence of trials. Proposition 1 states that whenever X has non-zero probability, however small, the probability that an X-world will be chosen at some point is 1.

Proposition 1 (van Fraassen [41]) *If* $\Pr(X) > 0$, *then the set of sequences containing an* X*-world has probability* 1.

Proof
$$\begin{aligned}
&\Pr^*(\{\omega^* \in \Omega^* | \omega^*[n] \in X \text{ for some } n \geqslant 1\})\\
&= 1 - \Pr^*(\{\omega^* \in \Omega^* | \omega^*[n] \notin X \text{ for all } n \geqslant 1\})\\
&= 1 - \lim_{n\to\infty} \Pr(\overline{X})^n\\
&= \begin{cases} 1 \text{ if } \Pr(X) > 0\\ 0 \text{ otherwise}\end{cases}
\end{aligned}$$
□

Proposition 1 is useful because it will allow us in our calculations to ignore those sequences which do not contain any X-worlds, provided that X has positive probability (see van Fraassen [41], for more details). What this means and why it is helpful will become clear momentarily.

First, however, an assignment V^* of truth values to sentences at world sequences in Ω^*, is again defined in terms of the assignment $\mathrm{V}_{\mathcal{A}}$ to atomic sentences — this time, however, for all of $\mathcal{L}_{\mathcal{A}}$ directly, not only $\mathcal{L}^0_{\mathcal{A}}$. The values of atomic sentences at ω^* are determined by $\omega^*[1]$. Conjunction and negation are interpreted as usual. The idea behind the treatment of conditionals is this: If the antecedent is true at ω^*, then the value of the conditional is that of its consequent at ω^*. Otherwise, its value is that of the consequent at the sequence obtained by "lopping off" worlds from ω^*, starting at $\omega^*[1]$, until a point is reached at which the antecedent is true. Proposition 1 assures us that the probability that this lopping-off continues infinitely, and thus the value of the conditional is undefined, is 0 whenever the antecedent has positive probability.

Definition 6 (Stalnaker Bernoulli model) Let $\langle \Omega^*, \mathcal{F}^*, \Pr^* \rangle$ be a Stalnaker Bernoulli space and $\mathrm{V}_{\mathcal{A}}$ a truth assignment to the atomic sentences in $\mathcal{A}$ at worlds in Ω. A *Stalnaker Bernoulli model for* $\mathcal{L}_{\mathcal{A}}$ is a quadruple $\langle \Omega^*, \mathcal{F}^*, \Pr^*, \mathrm{V}^* \rangle$, where V^* is an assignment of truth values to sentences in $\mathcal{L}_{\mathcal{A}}$, defined as follows, for all $\varphi, \psi \in \mathcal{L}_{\mathcal{A}}$ and $\omega^* \in \Omega^*$:

$$\begin{aligned}
\text{If } \varphi \in \mathcal{A}, \text{ then } \mathrm{V}^*(\varphi)(\omega^*) &= V_{\mathcal{A}}(\varphi)(\omega^*[1])\\
\mathrm{V}^*(\overline{\varphi})(\omega^*) &= 1 - \mathrm{V}^*(\varphi)(\omega^*)\\
\mathrm{V}^*(\varphi\psi)(\omega^*) &= \mathrm{V}^*(\varphi)(\omega^*) \times \mathrm{V}^*(\psi)(\omega^*)\\
\mathrm{V}^*(\varphi \to \psi)(\omega^*) &= \mathrm{V}^*(\psi)(\omega^*(n)) \text{ for the least } n\\
&\qquad \text{such that } \mathrm{V}^*(\varphi)(\omega^*(n)) = 1
\end{aligned}$$

Proposition 2 *If* $\varphi \in \mathcal{L}^0_{\mathcal{A}}$, *then* $\mathrm{V}^*(\varphi)(\omega^*) = \mathrm{V}(\varphi)(\omega^*[1])$.

Proof From Definitions 3 and 6, by induction on φ. □

In addition to Proposition 2, the following consequences of Definition 6 are worth noting: First, if the value of a conditional at a world sequence is defined, it is either 1 or 0, not some third value or something in between. This is why the remaining clauses of the definition apply to conditionals without special provisions. Second, the values of conditionals are undefined at sequences which do not contain any subsequences at which the antecedent is true. Third, no special provisions are required for the case that φ or ψ contain conditionals.

It is with respect to the second point that Proposition 1 is of great use. For any conditional whose antecedent has positive probability, the probability that its truth value is defined at a randomly chosen sequence is 1. Therefore the probability of the conditional can be calculated as the conditional expectation of its truth value, given that its truth value is defined. Throughout this paper, I will make the assumption that the antecedents of the conditionals I discuss have non-zero probability, hence that their truth values are defined with probability 1.

2.4 First-order Conditionals

For every probability model $\mathcal{M}$, there is a corresponding Stalnaker Bernoulli model $\mathcal{M}^*$ linked to $\mathcal{M}$ by the definition of the product measure and a common valuation $V_{\mathcal{A}}$ underlying both truth assignments. However, while the value assignment is defined for $\mathcal{L}_{\mathcal{A}}$ in $\mathcal{M}^*$, it is restricted to $\mathcal{L}_{\mathcal{A}}^0$ in $\mathcal{M}$. In the following, the correspondence between probability models and the Stalnaker Bernoulli models based on them is exploited in two steps: First, the probabilities of conditionals in $\mathcal{M}^*$, defined in Definition 7 below, can be calculated from the values they receive according to Definition 6 and certain probabilities in $\mathcal{M}$. Second, their values in $\mathcal{M}$ are defined in terms of their probabilities in $\mathcal{M}^*$. Ultimately, therefore, the reference to $\mathcal{M}^*$ can be omitted altogether and the values of all sentences in $\mathcal{M}$ can be defined with reference to probabilities in $\mathcal{M}$ alone. In this subsection, I define the requisite notions and illustrate with the relatively simple example of *first-order conditionals* — that is, sentences of the form '$A \rightarrow C$', where both A and C are in $\mathcal{L}_{\mathcal{A}}^0$.

Recall that Definition 4 above gave the probabilities of sentences in $\mathcal{L}_{\mathcal{A}}^0$ with reference to V and Pr. In a similar way, we can define the probabilities of sentences in $\mathcal{L}_{\mathcal{A}}$ in terms of V^* and Pr^*.

Definition 7 (Probabilities of sentences) The probability $P^*(\varphi)$ of a sentence φ in a model $\langle \Omega^*, \mathcal{F}^*, Pr^*, V^* \rangle$ is the expectation of its truth value:

$$P^*(\varphi) =_{df} E^*[V^*(\varphi)] = Pr^*(\{\omega^* \in \Omega^* | V^*(\varphi)(\omega^*) = 1\})$$

The right-hand side in Definition 7 is simpler than in Definition 4 above because the value of $V^*(\varphi)$ is guaranteed to be in $\{0, 1\}$ whenever it is defined at all. (Recall that the probability is implicitly conditionalized upon the event that $V^*(\varphi)$ is defined.)

How, then, are the probabilities assigned by P^* related to those assigned by P in the corresponding "unstarred" probability model? If φ is in $\mathcal{L}^0_{\mathcal{A}}$, the answer is simple:

Proposition 3 *If* $\varphi \in \mathcal{L}^0_{\mathcal{A}}$, *then* $P^*(\varphi) = P(\varphi)$.

Proof

$$\begin{aligned}
& \mathrm{P}^*(\varphi) = \mathrm{Pr}^*(\{\omega^* \in \Omega^* | V^*(\varphi)(\omega^*) = 1\}) \text{ by the definition of } P^* \\
(*) \quad & = \mathrm{Pr}^*(\{\omega^* \in \Omega^* | \mathrm{V}(\varphi)(\omega^*[1]) = 1\}) \text{ by Proposition 2} \\
(**) \quad & = \mathrm{Pr}^*(\{\omega \in \Omega | \mathrm{V}(\varphi)(\omega) = 1\} \times \Omega^*) \\
& = \mathrm{Pr}(\{\omega \in \Omega | V(\varphi)(\omega) = 1\}) \text{ by the definition of } \mathrm{Pr}^* \\
& = \mathrm{P}(\varphi)
\end{aligned}$$

□

More interesting is the case of a first-order conditional $A \to C$. Here the transition from (*) to (**) in the proof of Proposition 3 is not valid: We cannot count on there being a set X of worlds such that $\mathrm{V}^*(A \to C)(\omega^*)$ is true if and only if $\omega^*[1]$ is an X-world. Intuitively, this is because the set of sequences at which $\mathrm{V}^*(A \to C)$ is true "cuts across" classes of sequences sharing an initial world at which A is false. However, $\mathrm{P}^*(A \to C)$ can be calculated from probabilities already defined in $\langle \Omega, \mathcal{F}, \mathrm{Pr}, \mathrm{V} \rangle$ in a straightforward manner. To show this, the following lemma is useful.

Lemma 1 (Fraction Lemma—van Fraassen [41], p. 294) *For all* $X \in \mathcal{F}$ *such that* $\mathrm{Pr}(X) > 0$, $\sum_{i=0}^{\infty} \mathrm{Pr}(\overline{X})^i = 1/\mathrm{Pr}(X)$.

Proof

$$\begin{aligned}
& \left(\textstyle\sum_{i=0}^{\infty} \mathrm{Pr}(\overline{X})^i\right) \times \mathrm{Pr}(X) = \textstyle\sum_{i=0}^{\infty} \left(\mathrm{Pr}(\overline{X})^i \times \mathrm{Pr}(X)\right) \\
& = \textstyle\sum_{i=0}^{\infty} \mathrm{Pr}^*(\{\omega^* \in \Omega^* | \omega^*[n] \notin X \text{ for all } n \leqslant i \text{ and } \omega^*[i+1] \in X\}) \\
& \quad \text{by the definition of } \mathrm{Pr}^* \\
& = \mathrm{Pr}^*(\{\omega^* \in \Omega^* | \omega^*[n] \in X \text{ for some } n \geqslant 1\}) \\
& \quad \text{since the events in the summation are mutually disjoint} \\
& = 1 \text{ by Proposition 1 and since } \mathrm{Pr}(X) > 0.
\end{aligned}$$

□

With the help of Lemma 1, we obtain $\mathrm{P}^*(A \to C)$ as follows:

Theorem 1 *If* $A, C \in \mathcal{L}^0_{\mathcal{A}}$ *and* $\mathrm{P}(A) > 0$, *then* $\mathrm{P}^*(A \to C) = \mathrm{P}(C|A)$.

Proof At sequences ω^* at which A is true, the conditional is equivalent to the consequent. Thus:

$$\begin{aligned}
\text{(i)} \quad & \mathrm{P}^*((A \to C) \wedge A) = \mathrm{P}^*(AC) \\
\text{(ii)} \quad & = \mathrm{P}(AC) \text{ by Proposition 3 since } AC \in \mathcal{L}^0_{\mathcal{A}}
\end{aligned}$$

The step from (i) to (ii) will be crucial in the derivations below, where I will generally take it without further comment.

If A is false at ω^*, then $V^*(A \to C)(\omega^*)$ depends on the first sub-sequence at which A is true. Its probability in this case can be calculated using Theorem 1: For each $n > 1$, the conditional is true (false) at all sequences ω^* such that A is false at all $\omega^*(m)$, $m < n$, and AC $(A\overline{C})$ is true at $\omega^*(n)$. The expectation of these values is the following:

(iii) $$P^*((A \to C) \wedge \overline{A}) = P(\overline{A})P(AC) + P(\overline{A})^2 P(AC) + \ldots$$

The cases covered by (i) and (iii) are mutually exclusive and jointly exhaust the possible values of the antecedent A. Thus the overall probability of the conditional is as in (iv):[5]

(iv) $$\begin{aligned} P^*(A \to C) &= P(AC) + P(\overline{A})P(AC) + P(\overline{A})^2 P(AC) + \ldots \\ &= P(AC)/P(A) \text{ by Lemma 1} \\ &= P(C|A) \end{aligned}$$

□

Finally, given the probability $P^*(\varphi)$ of an arbitrary sentence φ, we can extend the assignment V of values at worlds in Ω. Intuitively, the value of a sentence φ at $\omega \in \Omega$ is the conditional expectation of $V^*(\varphi)$, given that the first world in the sequence is ω. Now, this conditional expectation is only defined if ω has non-zero probability. The countability of the probability space ensures that this is the case for some, though not all ω. A definition for the general case is given in a footnote.[6] The more restricted Definition 8 conveys the main idea. Here '[1]' functions as a random variable with domain Ω^* and range Ω.

Definition 8 (Assignment for $\mathcal{L}_{\mathcal{A}}$) For all $\omega \in \Omega$ with $\Pr(\omega) > 0$ and $\varphi \in \mathcal{L}_{\mathcal{A}}$:

$$\begin{aligned} V(\varphi)(\omega) &=_{df} E^*[V^*(\varphi)|[1] = \omega] \\ &= \Pr^*(\{\omega^* \in \Omega^* | V^*(\varphi)(\omega^*) = 1\} | \{\omega\} \times \Omega^*) \end{aligned}$$

This is equivalent to the conditional probability on the second line because the range of $V^*(\varphi)$, where defined, is $\{0, 1\}$, regardless of whether φ contains conditionals or not. The value is undefined if the probability of the sentence is undefined. In contrast to $V^*(\varphi)$, the range of $V(\varphi)$, where defined, is the interval $[0, 1]$, since the expectation of $V^*(\varphi)$ may fall anywhere between 0 and 1 if φ contains conditionals. Only for $\varphi \in \mathcal{L}^0_{\mathcal{A}}$ is $V(\varphi)$ guaranteed to be in $\{0, 1\}$.

[5] Lemma 1 is a statement about the probabilities of sets in $\mathcal{F}$, not sentences, but it is safe to refer to it here. By assumption, $\mathcal{F}$ is the full powerset of Ω, therefore all sentences in $\mathcal{L}^0_{\mathcal{A}}$ are guaranteed to denote (the characteristic functions of) subsets of Ω whose probabilities are defined.

[6] For the general case, we may follow Stalnaker and Jeffrey [40] and discard the first position of the sequences in question: $V(\varphi)(\omega) =_{df} \Pr^*(\{\omega^*(2) \in \Omega^* | V^*(\varphi)(\omega^*) = 1 \text{ and } \omega^*[1] = \omega\})$.

In fact, it is easy to check that for $\varphi \in \mathcal{L}^0_{\mathcal{A}}$, the assignment defined here satisfies the conditions in Definition 3 above (cf. also the proof of Proposition 3).

Consider, however, a first-order conditional $A \to C$. What are the values assigned to this sentence at worlds ω in Ω by the function V as defined here? The answer is already implicit in the proof of Theorem 1: At worlds ω at which the antecedent is true, the conditional is equivalent to its consequent. If, on the other hand, the antecedent is false, then its value is the expectation over the whole space Ω^* (as is also evident in the second line in Definition 8).

The last step is to show that this assignment indeed yields the intended probability.

Proposition 4 *The probability in Theorem* 1 *is the expectation of the values assigned to worlds in* Ω *according to the following rule:*

$$\mathrm{V}(A \to C)(\omega) = \begin{cases} 1 \textit{ if } \mathrm{V}(AC)(\omega) = 1 \\ 0 \textit{ if } \mathrm{V}(A\overline{C})(\omega) = 1 \\ \mathrm{P}(C|A) \textit{ if } \mathrm{V}(\overline{A})(\omega) = 1 \end{cases}$$

Proof
$$\begin{aligned} \mathrm{P}(A \to C) &= 1 \times \mathrm{P}(AC) + 0 \times \mathrm{P}(A\overline{C}) + \mathrm{P}(C|A)\mathrm{P}(\overline{A}) \\ &= \mathrm{P}(C|A)[\mathrm{P}(A) + \mathrm{P}(\overline{A})] = \mathrm{P}(C|A) \end{aligned}$$
□

First-order conditionals are the simplest case. Values for more complex sentences are also given by the above definitions. In the following section, I will show what these values are in the cases of interest in this paper.

3 Compounds of Conditionals

I now turn to the predictions of the account for embeddings and compounds of conditionals. Throughout this section, I will keep up the convention of using Roman letters (A, B, etc.) to refer to sentences in $\mathcal{L}^0_{\mathcal{A}}$, i.e., not containing conditionals. In principle, the definitions support arbitrary embeddings. However, I am not aware of any reports of intuitions about more complex sentences than the ones dealt with here, and since the purpose of this paper is to bring the approach in line with such intuitions, I limit the discussion to these cases.

The strategy is as in the first-order case above: Given a sentence φ of a certain form and a probability model $\mathcal{M}$, we can deduce the values of $\mathrm{V}(\varphi)(\omega)$ assigned to the sentence at worlds in $\mathcal{M}$ from the values $\mathrm{V}^*(\varphi)(\omega^*)$ assigned at world sequences in the Stalnaker Bernoulli model based on $\mathcal{M}$. I will go through this argument for each of four types of complex sentences, in each case showing that $\mathrm{P}(\varphi)$, the expectation of $\mathrm{V}(\varphi)$, equals $\mathrm{P}^*(\varphi)$, the expectation of $\mathrm{V}^*(\varphi)$.

3.1 Conjoined Conditionals

The first case I want to consider is that of conjoined conditionals, exemplified by (3a) and of the general form (3b).

(3) a. If Martha is in the kitchen, we'll have dinner soon, and if Marv is in the garage, the car will be fixed tonight.
b. $(A \to B) \wedge (C \to D)$

The values assigned to such a sentence at world sequences according to the rule for conjunction are given in (4).

(4) $$V^*((A \to B) \wedge (C \to D))(\omega^*) = V^*(A \to B)(\omega^*) \times V^*(C \to D)(\omega^*)$$

This evaluates to 1 if and only if B is true at the first A-subsequence in ω^* and D is true at the first C-subsequence in ω^*. Notice that since it is not required that the relevant subsequences be the same, the conditional may have non-zero probability even if A and C are mutually incompatible.

Theorem 2 *For all* A, B, C, D *such that* $P(A) > 0$ *and* $P(C) > 0$*:*

$$P^*((A \to B) \wedge (C \to D)) = \frac{P(ABCD) + P(D|C)P(AB\overline{C}) + P(B|A)P(CD\overline{A})}{P(A \vee C)}$$

Proof The simplest case is that in which the value of the conjunction is determined by the first world in the sequence of evaluation: The conjunction is true whenever A, B, C and D are all true, and it is false if one of the conjuncts is, i.e., if either A is true and B is false, or C is true and D is false, or both:

(i) $$P^*(((A \to B) \wedge (C \to D)) \wedge ABCD) = P(ABCD)$$

(ii) $$P^*(((A \to B) \wedge (C \to D)) \wedge A\overline{B}) = 0$$

(iii) $$P^*(((A \to B) \wedge (C \to D)) \wedge C\overline{D}) = 0$$

Next, consider the "mixed" case of an initial world w at which one conjunct is true and the other has a false antecedent. This is the case, for instance, where A and B are both true and C is false. At such sequences, the conjunction is true if D is true at the first C-subsequence (iv). Similarly for the case that C and D are true and A is false (v):

(iv) $$\begin{aligned} &P^*(((A \to B) \wedge (C \to D)) \wedge AB\overline{C}) \\ &= P(AB\overline{C})P(CD) + P(AB\overline{C})P(\overline{C})P(CD) \\ &\quad + P(AB\overline{C})P(\overline{C})^2P(CD) + \ldots \\ &= P(AB\overline{C})P(CD)/P(C) = P(D|C)P(AB\overline{C}) \end{aligned}$$

(v) $$P^*(((A \to B) \wedge (C \to D)) \wedge \overline{A}CD) = P(B|A)P(\overline{A}CD)$$

The cases in (i) through (v) jointly exhaust the event that $A \vee C$ is true. Thus:

(vi) $$\mathrm{P}^*(((A \rightarrow B) \wedge (C \rightarrow D)) \wedge (A \vee C))$$
$$= \mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(CD\overline{A})$$

Finally, in case both A and C are false, the conjunction is true at those sequences in which B is true at the first A-subsequence and D is true at the first C-subsequence. For each n, the set of sequences ω^* for which the relevant subsequence is $\omega^*(n)$ falls into three disjoint classes, each being of one of the types discussed so far: Either $ABCD$ is true at $\omega^*(n)$, or AB is true at $\omega^*(n)$ and CD later, or CD is true at $\omega^*(n)$ and AB later. Furthermore, for each n these three sets are mutually disjoint. Thus:

(vii) $$\mathrm{P}^*((A \rightarrow B) \wedge (C \rightarrow D))$$
$$= \text{(vi)} + \mathrm{P}(\overline{A}\,\overline{C}) \times \text{(vi)} + \mathrm{P}(\overline{A}\,\overline{C})^2 \times \text{(vi)} + \ldots$$
$$= \frac{\mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(CD\overline{A})}{\mathrm{P}(A \vee C)}$$

□

The proof suggests a way of assigning values to conjoined conditionals at worlds in Ω as follows.

Proposition 5 *The probability in Theorem* 2 *is the expectation of the values assigned at worlds in* Ω *according to the following rule:*

$$\mathrm{V}((A \rightarrow B) \wedge (C \rightarrow D))(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(ABCD)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(A\overline{B})(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(C\overline{D})(\omega) = 1 \\ \mathrm{P}(D|C) \text{ if } \mathrm{V}(AB\overline{C})(\omega) = 1 \\ \mathrm{P}(B|A) \text{ if } \mathrm{V}(\overline{A}CD)(\omega) = 1 \\ \text{(vii) if } \mathrm{V}(\overline{A}\,\overline{C})(\omega) = 1 \end{cases}$$

Proof

$$\mathrm{P}((A \rightarrow B) \wedge (C \rightarrow D))$$
$$= 1 \times \mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD)$$
$$+ \text{(vii)} \times \mathrm{P}(\overline{A}\,\overline{C})$$
$$= \text{(vii)} \times [\mathrm{P}(A \vee C) + \mathrm{P}(\overline{A}\,\overline{C})] = \text{(vii)}$$

□

3.2 Conditional Conditionals

Consider next conditionals both of whose constituents are conditionals, exemplified in (5a) and of the general form (5b).

(5) a. If this vase will crack if it is dropped on wood, it will break if it is dropped on granite.

b. $(A \to B) \to (C \to D)$

The values for sentences of this type at world sequences are assigned according to the following rule:

(6)
$$\begin{aligned} &\mathrm{V}^*((A \to B) \to (C \to D))(\omega^*) \\ &\qquad = \mathrm{V}^*(C \to D)(\omega^*(n)) \text{ for the least } n \text{ such that} \\ &\qquad\quad \mathrm{V}^*(A \to B)(\omega^*(n)) = 1 \end{aligned}$$

Theorem 3 *For all* A, B, C, D *such that* $\mathrm{P}(A) > 0$, $\mathrm{P}(C) > 0$ *and* $\mathrm{P}(B|A) > 0$*:*

$$\begin{aligned} &\mathrm{P}^*((A \to B) \to (C \to D)) \\ &\quad = \frac{\mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD)}{\mathrm{P}(A \vee C)\mathrm{P}(B|A)} \end{aligned}$$

Proof Consider first the sequences at which the antecedent $A \to B$ is true by virtue of both A and B being true at the first world. In this case, the value of the conditional is that of its consequent $C \to D$: true if either the first world is a CD-world (i), or else C is false at the first world and the first C-world is a D-world (ii). Furthermore, the conditional is false at these sequences if C is true and D false at the first world (iii). In (i) through (iii), recall that $\mathrm{V}^*((A \to B) \wedge A)(\omega^*) = \mathrm{V}^*(AB)(\omega^*)$ for all ω^*.

(i) $$\mathrm{P}^*(((A \to B) \to (C \to D)) \wedge (A \to B) \wedge ACD) = \mathrm{P}(ABCD)$$

(ii) $$\mathrm{P}^*(((A \to B) \to (C \to D)) \wedge (A \to B) \wedge A\overline{C}) = \mathrm{P}(D|C)\mathrm{P}(AB\overline{C})$$

(iii) $$\mathrm{P}^*(((A \to B) \to (C \to D)) \wedge (A \to B) \wedge AC\overline{D}) = 0$$

Consider next the sequences at which A is false and $A \to B$ and C are true. In this case, the value of the conditional depends on that of D: Where D is true, the value is as in (iv); where D is false, it is 0.

(iv)
$$\begin{aligned} &\mathrm{P}^*(((A \to B) \to (C \to D)) \wedge (A \to B) \wedge \overline{A}CD) \\ &\quad = \mathrm{P}(\overline{A}CD)\mathrm{P}(AB) + \mathrm{P}(\overline{A}CD)\mathrm{P}(\overline{A})\mathrm{P}(AB) \\ &\qquad + \mathrm{P}(\overline{A}CD)\mathrm{P}(\overline{A})^2\mathrm{P}(AB) + \ldots \\ &\quad = \mathrm{P}(\overline{A}CD)\mathrm{P}(AB)/\mathrm{P}(A) \\ &\quad = \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD) \end{aligned}$$

(v) $$\mathrm{P}^*(((A \to B) \to (C \to D)) \wedge (A \to B) \wedge \overline{A}C\overline{D}) = 0$$

The cases in (i) through (v) are mutually exclusive and jointly exhaust all sequences at which both $A \vee C$ and the antecedent $A \rightarrow B$ are true, thus:

(vi)
$$\begin{aligned} &\mathrm{P}^*(((A \rightarrow B) \rightarrow (C \rightarrow D)) \wedge (A \rightarrow B) \wedge (A \vee C)) \\ &\quad = \mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD) \end{aligned}$$

Finally, at $\overline{A}\,\overline{C}$-sequences at which $A \rightarrow B$ is true, the value of the conditional is that of (vi) at the first world at which either A or C is true. (There is such a world in the sequence, for otherwise $A \rightarrow B$ would not be true.) Thus the probability that both the conditional and its antecedent are true is derived as follows:

(vii)
$$\begin{aligned} &\mathrm{P}^*(((A \rightarrow B) \rightarrow (C \rightarrow D)) \wedge (A \rightarrow B)) \\ &\quad = \text{(vi)} + \mathrm{P}(\overline{A}\,\overline{C}) \times \text{(vi)} + \mathrm{P}(\overline{A}\,\overline{C})^2 \times \text{(vi)} + \ldots \\ &\quad = \frac{\mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD)}{\mathrm{P}(A \vee C)} \end{aligned}$$

This is, not surprisingly, the probability of the conjunction $(A \rightarrow B) \wedge (C \rightarrow D)$.[7] From it, we can calculate the overall probability of the conditional as in (viii) (recall that $\mathrm{P}(A \rightarrow B) = \mathrm{P}(B|A)$):

(viii)
$$\begin{aligned} &\mathrm{P}^*((A \rightarrow B) \rightarrow (C \rightarrow D)) \\ &\quad = \text{(vii)} + \mathrm{P}(\overline{A \rightarrow B}) \times \text{(vii)} + \mathrm{P}(\overline{A \rightarrow B})^2 \times \text{(vii)} + \ldots \\ &\quad = \text{(vii)}/\mathrm{P}(A \rightarrow B) \\ &\quad = \frac{\mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD)}{\mathrm{P}(A \vee C)\mathrm{P}(B|A)} \end{aligned}$$

□

To derive the corresponding assignment of values to the conditional at worlds in Ω, we need to distinguish more cases than before. Where AB is true, the case is clear, and at $A\overline{B}$-worlds, the value is the overall probability (viii). Hence the first four lines in Proposition 6. Where A is false, the antecedent is true with probability $\mathrm{P}(B|A)$, and in this case the value of the conditional is that of $C \rightarrow D$. Where A and $A \rightarrow B$ are both false, the value is that of $C \rightarrow D$ at the first world at which $A \rightarrow B$ is true.

[7] This is not surprising for the following reason: (i) In the product space, $\mathrm{P}^*((A \rightarrow B) \rightarrow (C \rightarrow D)) = \mathrm{P}^*(C \rightarrow D|A \rightarrow B) = \mathrm{P}^*((A \rightarrow B) \wedge (C \rightarrow D))/\mathrm{P}^*(A \rightarrow B)$. (ii) Furthermore, the Thesis implies that the conditional and its antecedent are stochastically independent [41]; hence, $\mathrm{P}^*(((A \rightarrow B) \rightarrow (C \rightarrow D)) \wedge (A \rightarrow B)) = \mathrm{P}^*((A \rightarrow B) \rightarrow (C \rightarrow D))\mathrm{P}^*(A \rightarrow B)$, which by (i) equals $\mathrm{P}^*((A \rightarrow B) \wedge (C \rightarrow D))$.

Proposition 6 *The probability in Theorem 3 is the expectation of the values assigned at worlds in Ω according to the following rule:*

$$\mathrm{V}((A \to B) \to (C \to D))(\omega)$$
$$= \begin{cases} 1 \text{ } \textit{if} \text{ } \mathrm{V}(ABCD)(\omega) = 1 \\ 0 \text{ } \textit{if} \text{ } \mathrm{V}(ABC\overline{D})(\omega) = 1 \\ \mathrm{P}(D|C) \text{ } \textit{if} \text{ } \mathrm{V}(AB\overline{C})(\omega) = 1 \\ \text{(viii) } \textit{if} \text{ } \mathrm{V}(A\overline{B})(\omega) = 1 \\ \mathrm{P}(B|A) + \text{(viii)} \times \mathrm{P}(\overline{B}|A) \text{ } \textit{if} \text{ } \mathrm{V}(\overline{A}CD)(\omega) = 1 \\ \text{(viii)} \times \mathrm{P}(\overline{B}|A) \text{ } \textit{if} \text{ } \mathrm{V}(\overline{A}C\overline{D})(\omega) = 1 \\ \text{(viii) } \textit{if} \text{ } \mathrm{V}(\overline{A}\,\overline{C})(\omega) = 1 \end{cases}$$

Proof

$$\begin{aligned} &\mathrm{P}\ ((A \to B) \to (C \to D)) \\ &= \mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \text{(viii)} \times \mathrm{P}(A\overline{B}) \\ &\quad + [\mathrm{P}(B|A) + \text{(viii)} \times \mathrm{P}(\overline{B}|A)]\mathrm{P}(\overline{A}CD) \\ &\quad + \text{(viii)} \times \mathrm{P}(\overline{B}|A)\mathrm{P}(\overline{A}C\overline{D}) + \text{(viii)} \times \mathrm{P}(\overline{A}\,\overline{C}) \\ &= \mathrm{P}(ABCD) + \mathrm{P}(D|C)\mathrm{P}(AB\overline{C}) + \mathrm{P}(B|A)\mathrm{P}(\overline{A}CD) \\ &\quad + \text{(viii)} \times \mathrm{P}(\overline{B}|A) \times [\mathrm{P}(A) + \mathrm{P}(\overline{A}CD) + \mathrm{P}(\overline{A}C\overline{D})] \\ &\quad + \text{(viii)} \times \mathrm{P}(\overline{A}\,\overline{C}) \\ &= \text{(viii)} \times \mathrm{P}(B|A)\mathrm{P}(A \vee C) + \text{(viii)} \times \mathrm{P}(\overline{B}|A)\mathrm{P}(A \vee C) \\ &\quad + \text{(viii)} \times \mathrm{P}(\overline{A}\,\overline{C}) \\ &= \text{(viii)} \times [\mathrm{P}(A \vee C) + \mathrm{P}(\overline{A}\,\overline{C})] = \text{(viii)} \end{aligned}$$

□

3.3 Right-nested Conditionals

Next, I turn to right-nested conditionals. Their treatment is straightforward at this point because they are special cases of conditional conditionals. An example is (7a).

(7) a. If the match is wet, then if it is struck, it will light.
b. $B \to (C \to D)$

The values of such sentences at world sequences in Ω^* are defined as follows.

(8) $$\mathrm{V}^*(B \to (C \to D))(\omega^*) = \mathrm{V}^*(C \to D)(\omega^*(n)) \text{ for the least } n \text{ such that } \mathrm{V}^*(B)(\omega^*(n)) = 1$$

Theorem 4 *For all B, C, D such that* $\mathrm{P}(B) > 0$ *and* $\mathrm{P}(C) > 0$*:*

$$\mathrm{P}^*(B \to (C \to D)) = \mathrm{P}(CD|B) + \mathrm{P}(D|C)\mathrm{P}(\overline{C}|B)$$

Proof The antecedent B is equivalent to the conditional $\top \to B$, where $\top$ stands for an arbitrary tautology. Thus the sentence is a special case of the conditional conditionals discussed in Section 3.2 above:

$$\text{(i)} \quad \mathrm{P}^*((\top \to B) \to (C \to D)) = \frac{\mathrm{P}(\top BCD) + \mathrm{P}(D|C)\mathrm{P}(\top B\overline{C}) + \mathrm{P}(B|\top)\mathrm{P}(CD\overline{\top})}{\mathrm{P}(\top \vee C)\mathrm{P}(B|\top)}$$

$$\text{(ii)} \quad = \frac{\mathrm{P}(BCD) + \mathrm{P}(D|C)\mathrm{P}(B\overline{C})}{\mathrm{P}(B)}$$

$$= \mathrm{P}(CD|B) + \mathrm{P}(D|C)\mathrm{P}(\overline{C}|B)$$

□

Proposition 7 *The probability in Theorem 4 is the expectation of the values assigned at worlds in Ω according to the following rule:*

$$\mathrm{V}(B \to (C \to D))(\omega) = \begin{cases} 1 \textit{ if } \mathrm{V}(BCD)(\omega) = 1 \\ 0 \textit{ if } \mathrm{V}(BC\overline{D})(\omega) = 1 \\ \mathrm{P}(D|C) \textit{ if } \mathrm{V}(B\overline{C})(\omega) = 1 \\ \text{(i) } \textit{if } \mathrm{V}(\overline{B})(\omega) = 1 \end{cases}$$

Proof Immediate from Proposition 6, with A a tautology. □

Incidentally, the formula in Theorem 4 is also obtained, via a somewhat different route, by Jeffrey [20]. It is worth noting that under this assignment, the probabilistic *Import-Export Principle*, according to which $\mathrm{P}(B \to (C \to D))$ is equivalent to $\mathrm{P}(BC \to D) = \mathrm{P}(D|BC)$, is not valid. Notice that this principle would have followed if we had assigned $\mathrm{P}(D|BC)$ instead of $\mathrm{P}(D|C)$ at those worlds at which B is true and C is false. In this case, the right-hand side in the proof would be (ii′) instead of (ii), and as a consequence, the overall probability of the sentence would be $\mathrm{P}(D|BC)$.

$$\text{(ii}'\text{)} \quad [\mathrm{P}(BCD) + \mathrm{P}(D|BC)\mathrm{P}(B\overline{C})]/\mathrm{P}(B)$$

$$= [\mathrm{P}(D|BC)\mathrm{P}(BC) + \mathrm{P}(D|BC)\mathrm{P}(B\overline{C})]/\mathrm{P}(B)$$

$$= \mathrm{P}(D|BC)$$

Although the probabilistic Import–Export Principle has its staunch adherents [12, 30], it has often been argued that it does not hold in general. Adams [3] notes that it would be a mistake to postulate it as a general principle. Kaufmann [22] argues, in the same vein, that while it is often intuitively the case that $\mathrm{P}(B \to (C \to D))$ equals $\mathrm{P}(BC \to D)$, such judgments actually rest on independent additional intuitions about *causal* relations.

3.4 Left-nested Conditionals

Finally, consider conditionals with conditional antecedents, exemplified in (9a) and of the general form (9b).

(9) a. If this switch will fail if it is submerged in water, it will be discarded.

b. $(A \to B) \to D$

The values for this conditional are assigned according to the rule in (10).

(10) $$\mathrm{V}^*((A \to B) \to D)(\omega^*) = \mathrm{V}^*(D)(\omega^*(n)) \text{ for the least } n \text{ such that } \mathrm{V}^*(A \to B)(\omega^*(n)) = 1$$

Theorem 5 *For all* A, B, D *such that* $\mathrm{P}(A) > 0$ *and* $\mathrm{P}(B|A) > 0$*:*

$$\mathrm{P}^*((A \to B) \to D) = \mathrm{P}(D|AB)\mathrm{P}(A) + \mathrm{P}(D\overline{A})$$

Proof Similarly to the right-nested case above, we can replace the consequent D with a conditional $\top \to D$ whose antecedent is a tautology. Thus:

(i) $$\begin{aligned} &\mathrm{P}^*((A \to B) \to (\top \to D)) \\ &= \frac{\mathrm{P}(AB\top D) + \mathrm{P}(D|\top)\mathrm{P}(AB\overline{\top}) + \mathrm{P}(B|A)\mathrm{P}(\top D\overline{A})}{\mathrm{P}(A \vee \top)\mathrm{P}(B|A)} \\ &= \frac{\mathrm{P}(ABD) + \mathrm{P}(B|A)\mathrm{P}(D\overline{A})}{\mathrm{P}(B|A)} \\ &= \mathrm{P}(D|AB)\mathrm{P}(A) + \mathrm{P}(D\overline{A}) \end{aligned}$$

□

Proposition 8 *The probability in Theorem 5 is the expectation of the values assigned at worlds in* Ω *according to the following rule:*

$$\mathrm{V}((A \to B) \to D)(\omega) = \begin{cases} 1 \text{ } \mathit{if} \text{ } \mathrm{V}(ABD)(\omega) = 1 \\ 0 \text{ } \mathit{if} \text{ } \mathrm{V}(AB\overline{D})(\omega) = 1 \\ (i) \text{ } \mathit{if} \text{ } \mathrm{V}(A\overline{B})(\omega) = 1 \\ \mathrm{P}(B|A) + (i) \times \mathrm{P}(\overline{B}|A) \text{ } \mathit{if} \text{ } \mathrm{V}(\overline{A}D)(\omega) = 1 \\ (i) \times \mathrm{P}(\overline{B}|A) \text{ } \mathit{if} \text{ } \mathrm{V}(\overline{A}\,\overline{D})(\omega) = 1 \end{cases}$$

Proof From Proposition 6, with C a tautology. □

4 Two Modifications

The last section outlined in detail the treatment of embedded and conjoined conditionals explored by van Fraassen [41], Jeffrey [20] and Stalnaker and

Jeffrey [40]. Van Fraassen developed the model theory based on the product space construction, Jeffrey arrived at similar formulas for conditionals via a different route, and Stalnaker and Jeffrey synthesized the approaches. McGee [30] independently proposed the formula in Section 3.1 for conjoined conditionals.[8] All of these accounts are vulnerable to the charge of making counterintuitive predictions, which was leveled at them most notably by Lance [25] and Edgington [8]. In the next section, I will discuss such counterexamples. First, however, by way of preview, I will introduce two modifications to the account which will be crucial in correcting the problems. The two amendments are (i) a revised rule for assigning values to conditionals at world sequences which is sensitive to *causal independencies*, and (ii) a distinction between what I will call, following Kaufmann [21], *local* and *global* interpretations of conditionals. I will discuss both of these in turn.

4.1 Causal Relations

The appeal to causal dependencies is originally motivated by the desire to give a sensible interpretation for the values assigned to conditionals at worlds at which their antecedents are false. Jeffrey and Stalnaker did not devote any significant discussion to the question of why conditionals should receive such non-standard values and how these values should be interpreted. This omission is among the major conceptual problems identified by Edgington [9], Hájek and Hall [14], and others.

4.1.1 Motivation

Kaufmann [22] proposes that the values at non-antecedent worlds be interpreted as those of the corresponding counterfactual. This requires a modification, however, since the assignment functions discussed so far do not naturally lend themselves to this interpretation. To see this, consider the sentences in (11).

(11) a. If the match is struck, it will light.
b. If the match had been struck, it would have lit.

Let the indicative conditional in (11a) be denoted by '$S \rightarrow L$', and consider its evaluation in a model in which the event that the match is wet has high probability. Suppose further that the match is likely to light if it is struck and dry, but unlikely to light if it is struck and wet. The overall conditional probability that it will light, given that it is struck, $P(L|S)$, is low since the

[8]McGee also proposed an account of right-nested conditionals, which is based on the Import-Export Principle and thus not equivalent to the one in Section 3.3. McGee did not deal with conditional antecedents.

match is most likely wet. According to the rules for assigning values to the conditional, the corresponding low value is assigned at *all* worlds at which the match is not struck.[9]

But this means that these values cannot be interpreted as those of the counterfactual in (11b). Speakers generally agree that whether the match would have lit if it had been struck does depend on whether it is wet. At those worlds at which it is wet, the counterfactual should receive a low value; in contrast, the value should be high at worlds at which it is dry. To put it differently, the value of the counterfactual depends on that of a different sentence, call it 'W', which is true if and only if the match is wet; and since W is not mentioned in the conditional itself, the assignment cannot depend solely on the values of its constituents and the probability distribution.

This is Kaufmann's [22] argument for introducing additional structure into the model, with the goal of making the value assignment sensitive to such structure.

4.1.2 Formal Representation

The basic idea is to encode information about the *causal independencies* inherent in the worlds in Ω. Formally, rather than complicating the internal structure of the worlds themselves, this information is represented as a relation between random variables with range $\{0, 1\}$. They are thus of the same type as the denotations $\mathrm{V}(\varphi)$ of sentences φ in $\mathcal{L}^0_{\mathcal{A}}$. There is no reason in principle to limit the representation to those variables that coincide with the interpretation of some sentence; any indicator function may enter into causal relations, not just those that happen to have names. However, for simplicity in the illustrations, I will only deal with variables that *are* equivalent to sentence interpretations, and I will represent them as '$\mathrm{V}(\varphi)$', where $\varphi \in \mathcal{L}^0_{\mathcal{A}}$ is the sentence in question.

Definition 9 (Causal probability space) A *causal probability space* is a quintuple $\langle \Omega, \mathcal{F}, \Pr, \Phi, \prec \rangle$, where $\langle \Omega, \mathcal{F}, \Pr \rangle$ is a probability model, Φ is a set of random variables in $\{0, 1\}^\Omega$, and $\prec$ is a strict partial order on Φ.[10]

Causal probability spaces give rise to the causal analogs of the structures defined in Section 2 above. I begin with the notion of a causal probability model.

---

[9]This and other examples are discussed in more detail in Kaufmann [22].

[10]The structure $\langle \Phi, \prec \rangle$ is the transitive closure of a *Directed Acyclic Graph* (DAG), the construct of choice in the burgeoning literature on causality in statistics, artificial intelligence, psychology and philosophy ([7, 13, 15, 32, 37, 38, 43], and others).

Definition 10 (Causal probability model) Let $\langle\Omega, \mathcal{F}, \Pr, \Phi, \prec\rangle$ be a causal probability space. A sextuple $\langle\Omega, \mathcal{F}, \Pr, \mathbb{V}, \Phi, \prec\rangle$ is a *causal probability model for* $\mathcal{L}^0_{\mathcal{A}}$ iff $\langle\Omega, \mathcal{F}, \Pr, \mathbb{V}\rangle$ is a probability model for $\mathcal{L}^0_{\mathcal{A}}$ (cf. Definition 3).

It may seem peculiar that the causal structure in a causal probability model does not affect the definition of the assignment function. However, this is only the case for sentences in $\mathcal{L}^0_{\mathcal{A}}$. For those, the causal structure can indeed be ignored. But ultimately the assignment will be extended to all sentences in $\mathcal{L}_{\mathcal{A}}$. The causal structure will then become relevant for the values of sentences containing conditionals, and the equivalence between the two valuation functions will break down.

A causal probability space is associated with a causal Stalnaker Bernoulli space as expected:

Definition 11 (Causal Stalnaker Bernoulli space) A *causal Stalnaker Bernoulli space* based on a causal probability space $\langle\Omega, \mathcal{F}, \Pr, \Phi, \prec\rangle$ is a structure $\langle\Omega^*, \mathcal{F}^*, \Pr^*, \Phi^*, \prec^*\rangle$, where

a. $\langle\Omega^*, \mathcal{F}^*, \Pr^*\rangle$ is the Stalnaker Bernoulli space based on $\langle\Omega, \mathcal{F}, \Pr\rangle$;
b. For each $X \in \{0, 1\}^{\Omega}$, let $X^* \in \{0, 1\}^{\Omega^*}$ be that function such that for all $\omega^* \in \Omega^*$, $X^*(\omega^*) = X(\omega^*[1])$. Then $\Phi^* = \{X^* | X \in \Phi\}$.
c. For all $X^*, Y^* \in \Phi^*$, $X^* \prec^* Y^*$ iff $X \prec Y$.

These definitions imply that the interpretations of conditionals do not stand in causal relations, since they do not in general coincide with (characteristic functions of) subsets of Ω. (They may happen to be equivalent to a variable in Φ in some models, but in that case I will assume that there is an equivalent truth-functional sentence.) Although it may be technically possible to allow for such dependencies, it is not clear to me whether such a move would be conceptually plausible. In any case, I will not pursue this possibility any further. As we will see later, this decision removes from the purview of this paper certain theoretically possible readings for conditionals with conditional antecedents. The proposal does not suffer from this limitation, however, because it is the remaining readings that I am concerned with.

In the causal model, sentences are assigned values at world sequences just as in Definition 6 above, save for the clause for conditionals. In order to set this new assignment apart from the function V^* defined earlier, I define a new assignment function '$\mathbb{V}^*$' which is sensitive to the causal structure in the model.

Definition 12 (Causal Stalnaker Bernoulli model) Let $\langle\Omega^*, \mathcal{F}^*, \Pr^*, \Phi^*, \prec^*\rangle$ be a causal Stalnaker Bernoulli space and $V_{\mathcal{A}}$ a truth assignment to the atomic sentences in $\mathcal{A}$ at worlds in Ω. A *causal Stalnaker Bernoulli model* for $\mathcal{L}_{\mathcal{A}}$ is a sextuple $\langle\Omega^*, \mathcal{F}^*, \mathrm{Pr}^*, \mathbb{V}^*, \Phi^*, \prec^*\rangle$, where $\mathbb{V}^*$ is an assignment of truth values to sentences in $\mathcal{L}_{\mathcal{A}}$, defined as follows, for all $\varphi, \psi \in \mathcal{L}_{\mathcal{A}}, \omega^* \in \Omega^*$:

$$\begin{aligned}
\text{If } \varphi \in \mathcal{A}, \mathbb{V}^*(\varphi)(\omega^*) =& V_{\mathcal{A}}(\varphi)(\omega^*[1]) \\
\mathbb{V}^*(\overline{\varphi})(\omega^*) =& 1 - \mathbb{V}^*(\varphi)(\omega^*) \\
\mathbb{V}^*(\varphi\psi)(\omega^*) =& \mathbb{V}^*(\varphi)(\omega^*) \times \mathbb{V}^*(\psi)(\omega^*) \\
\mathbb{V}^*(\varphi \to \psi)(w^*) =& \mathbb{V}^*(\psi)(\omega^*(n)) \text{ for the least } n \text{ such that:} \\
& \quad \mathbb{V}^*(\varphi)(\omega^*(n)) = 1 \text{ and} \\
& \qquad X^*(\omega^*(n)) = X^*(\omega^*) \text{ for all } X^* \in \Phi^* \\
& \qquad \text{such that } \mathbb{V}^*(\varphi) \npreceq^* X^*, \\
& \quad \text{if } \varphi \in \mathcal{L}^0_{\mathcal{A}}; \\
& \quad \mathbb{V}^*(\varphi)(\omega^*(n)) = 1, \text{ otherwise}
\end{aligned}$$

The added condition in the first part of the clause for conditionals only makes a difference for sequences at which the antecedent is false. The effect is that now the subsequence starting with the first antecedent-world will not necessarily determine the truth value of the conditional. Some of the antecedent-subsequences that were relevant for V^* are now discarded in the process of "lopping off" initial worlds. Only those remain "visible" which agree with ω^* on the values of those variables that are causally independent of the denotation of the antecedent. The condition in the last line reads '$\mathbb{V}^*(\varphi) \npreceq^* X^*$', rather than '$\mathbb{V}^*(\varphi) \nprec^* X^*$', because otherwise all antecedent-subsequences would be rendered irrelevant at a sequence at which the antecedent is false, if its denotation is among the causally relevant variables.

The clause for conditionals distinguishes two cases mainly for convenience. The first is restricted to antecedents not containing conditionals. The second covers antecedents which contain conditionals. These are treated as before. Since conditionals are exempt from causal dependencies, the variables in Φ^* have no effect on conditional antecedents.

For convenience of reference in subsequent sections, Definition 13 fixes some terminology relating causal and non-causal models which share the same probability space and truth assignment to atomic sentences.

Definition 13 (Causal extensions and non-causal counterparts) Let $M = \langle\Omega, \mathcal{F}, \mathrm{Pr}, \mathrm{V}\rangle$ and $\mathbb{M} = \langle\Omega, \mathcal{F}, \mathrm{Pr}, \mathbb{V}, \Phi, \prec\rangle$ be two models such that $\mathrm{V}(\varphi)(\omega) = \mathbb{V}(\varphi)(\omega)$ for all $\varphi \in \mathcal{A}, \omega \in \Omega$. Then $\mathbb{M}$ is a *causal extension of* M, and M is the *non-causal counterpart* of $\mathbb{M}$. Similarly for the Stalnaker Bernoulli models M^* and $\mathbb{M}^*$ based on M and $\mathbb{M}$, respectively.

While for each causal model there is exactly one non-causal counterpart, a given non-causal model has many causal extensions. In the remainder of the paper, I will contrast the values and probabilities of sentences in certain causal models with the corresponding values and probabilities in their non-causal counterparts. I will continue to distinguish the respective notions typographically, writing $\mathbb{V}^*$, $\mathbb{V}$, $\mathbb{P}^*$ and $\mathbb{P}$ for the causal model, corresponding to V^*, V, P^* and P under the non-causal interpretation, respectively. The probabilities $\mathbb{P}^*$

and $\mathbb{P}$ do not require new definitions, since they are determined by $\mathbb{V}^*/\mathbb{V}$ and $\mathrm{Pr}^*/\mathrm{Pr}$ as before (see Definitions 4 and 7 above).

Finally, the following facts, though not surprising, are nevertheless worth pointing out, as they will help simplify the exposition considerably.

Proposition 9 *Let* $M = \langle \Omega, \mathcal{F}, \mathrm{Pr}, \mathrm{V} \rangle$ *be a probability model,* $\mathbb{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr}, \mathrm{V}, \Phi, \prec \rangle$ *a causal extension of* M, M^* *and* $\mathbb{M}^*$ *the Stalnaker Bernoulli models based on* M *and* $\mathbb{M}$*, respectively, and* φ *an arbitrary sentence in* $\mathcal{L}^0_{\mathcal{A}}$*. Then*

a. *For all* $\omega \in \Omega$, $\mathrm{V}(\varphi)(\omega) = \mathbb{V}(\varphi)(\omega)$*; and*
b. *For all* $\omega^* \in \Omega^*$, $\mathrm{V}^*(\varphi)(\omega^*) = \mathbb{V}^*(\varphi)(\omega^*)$

Proof From Definitions 3, 6, 10, 12, by induction on φ. □

Proposition 10 *Let* M, $\mathbb{M}$, M^* *and* $\mathbb{M}^*$ *as in Proposition* 9, *and* φ *an arbitrary sentence in* $\mathcal{L}^0_{\mathcal{A}}$*. Then* $\mathrm{P}(\varphi) = \mathbb{P}(\varphi) = \mathrm{P}^*(\varphi) = \mathbb{P}^*(\varphi)$.

Proof $\mathrm{P}(\varphi) = \mathbb{P}(\varphi)$ and $\mathrm{P}^*(\varphi) = \mathbb{P}^*(\varphi)$ are corollaries of Proposition 9. $\mathrm{P}(\varphi) = \mathrm{P}^*(\varphi)$ by Proposition 3. □

Frequently in the formulas below, V and P will co-occur in the same formula with the corresponding $\mathbb{V}$ and $\mathbb{P}$. Propositions 9 and 10 ensure that this shift is safe as long as the sentence in question does not contain conditionals.

4.1.3 Causality and First-order Conditionals

Before turning to more complex sentences, I will illustrate the consequences of the new definitions for first-order conditionals, i.e., those whose main connective is the conditional and whose constituents do not contain conditionals. For simplicity, I limit the discussion to the case that only one causally relevant background variable enters the calculation. For concreteness I return to the sentence in (11) above, repeated here as (12).

(12) a. If the match is struck, it will light.
b. $S \rightarrow L$

Suppose the set Φ of relevant variables is $\{\mathrm{V}(W), \mathrm{V}(S), \mathrm{V}(L)\}$, the causal dependencies are $\mathrm{V}(W) \prec \mathrm{V}(L)$ and $\mathrm{V}(S) \prec \mathrm{V}(L)$, and no other variables stand in the '$\prec$'-relation. These causal dependencies carry over to the causal Stalnaker Bernoulli space as described above. Thus, according to Definition 12, the values assigned to the sentence in (12b) at world sequences in Ω^* are the following (recall that the substitution of V^* for $\mathbb{V}^*$ in is safe for sentences in $\mathcal{L}^0_{\mathcal{A}}$):

(13)
$$\mathbb{V}^*(S \to L)(\omega^*) = \begin{cases} \mathrm{V}^*(L)(\omega^*(n)) \text{ for the least } n \text{ such that} \\ \qquad \mathrm{V}^*(SW)(\omega^*(n)) = 1, \ \text{if } \mathrm{V}^*(W)(\omega^*) = 1; \\ \mathrm{V}^*(L)(\omega^*(n)) \text{ for the least } n \text{ such that} \\ \qquad \mathrm{V}^*(S\overline{W})(\omega^*(n)) = 1, \ \text{if } \mathrm{V}^*(\overline{W})(\omega^*) = 1 \end{cases}$$

In other words, the sentence is equivalent to (14a) if the match is wet at the first world of the sequence, and to (14b) if the match is dry at that world.

(14) a. If the match is wet and you strike it, it will light.
b. If the match is dry and you strike it, it will light.

This interpretation accords well with intuitions about the corresponding counterfactual (15), whose value, as I argued above, also depends on whether the match is wet or dry.

(15) If the match had been struck, it would have lit.

Most importantly for the present discussion, the revised value assignment yields a new probability for the indicative conditional (12), which can be written down in two different but equivalent ways as follows.

(16)
$$\begin{aligned}\mathbb{P}^*(S \to L) &= \mathrm{P}(SL) + \mathrm{P}(L|SW)\mathrm{P}(\overline{S}W) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{S}\,\overline{W}) \\ &= \mathrm{P}(L|SW)\mathrm{P}(W) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W})\end{aligned}$$

To see this, consider W-worlds and $\overline{W}$-worlds separately. The probability of the conditional is the sum of (17) and (18).[11]

(17)
$$\mathbb{P}^*((S \to L) \wedge W) = \mathrm{P}(L|SW)\mathrm{P}(W)$$

(18)
$$\mathbb{P}^*((S \to L) \wedge \overline{W}) = \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W})$$

Each of the formulas on the right-hand side of (16) highlights a different intuition about the interpretation of such sentences. The difference will be important below.

4.2 Local and Global Interpretations

The causally sensitive assignment in Definition 12 is motivated by intuitive judgments about counterfactuals. The probability of the *indicative* conditional $S \to L$ under this assignment was given in (16). Now, it is important to note

[11]
$$\begin{aligned}\mathbb{P}^*((S \to L) \wedge W) &= \mathrm{P}(SLW) + \mathrm{P}(\overline{S}W)\mathrm{P}(SLW) + \mathrm{P}(\overline{S}W)\mathrm{P}(\overline{SW})\mathrm{P}(SLW) \\ &\quad + \mathrm{P}(\overline{S}W)\mathrm{P}(\overline{SW})^2\mathrm{P}(SLW) + \ldots \\ &= \mathrm{P}(SLW) + \mathrm{P}(\overline{S}W)\mathrm{P}(SLW)/\mathrm{P}(SW) \\ &= \mathrm{P}(L|SW)[\mathrm{P}(SW) + \mathrm{P}(\overline{S}W)] = \mathrm{P}(L|SW)\mathrm{P}(W)\end{aligned}$$
The argument for (18) is similar.

that $\mathbb{P}^*(S \to L)$ is not equivalent to the conditional probability of L, given S (unlike $\mathrm{P}^*(S \to L)$, which *is* equivalent to the conditional probability). The latter is given in (19).

(19)
$$\begin{aligned} \mathrm{P}(L|S) &= \mathrm{P}(SL) + \mathrm{P}(L|S)\mathrm{P}(\overline{S}) \\ &= \mathrm{P}(L|SW)\mathrm{P}(W|S) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W}|S) \end{aligned}$$

The two lines on the right-hand side of (19) mirror those in (16) above. The first is the one familiar from the earlier sections of this paper. The second simplifies to the left-hand side via a few straightforward transformations.

On the face of it, the discrepancy between (16) and (19) would seem to reveal a troubling tension between the unified account of indicatives and counterfactuals on the one hand, and the central premise of the probabilistic account, on the other. However, Kaufmann [21] and Kaufmann et al. [23] argue that upon closer inspection, it turns out that the difference points to a real but rarely discussed variability in the interpretation of indicative conditionals. The assumption that indicative conditionals have *both* readings enables [21] to explain a number of purported counterexamples to the probabilistic account from the philosophical literature; furthermore, Kaufmann et al. [23] report experimental results showing that subjects' probability judgments exhibit a variability which is predicted under the assumption that both readings are available, but which would be unexplained otherwise.

The explanation of the difference, in the case of the "match" example, goes as follows. Suppose you are more likely to strike the match if it is dry than if it is wet. Then your striking it, if you do strike it, provides evidence that it is dry, even if there is no sense in which it objectively *makes* it more likely to be dry. Suppose further I believe that it is most likely wet. In one sense, then, I believe that the match (still) won't light (even) if you strike it. On the other hand, I believe that if you strike it, it is probably dry after all, so it will likely light. The former interpretation is "local," in the sense that the hypothesized striking of the match does not affect my beliefs about its wetness. The latter is a more "global" interpretation, in that I take into account the epistemic repercussions of the hypothesized truth of the antecedent for my beliefs about the wetness. The two readings are paraphrased in (20). In the present scenario, the probability of the conditional is low under the local interpretation and high under the global one.[12]

(20) a. (Even) if you strike it, it won't light. *[local]*
b. If you strike it, (it must be dry after all, so) it will light. *[global]*

The two versions of the formulas in each of (16) and (19) suggest two directions in which the difference between local and global readings may be

[12] The epistemic '*must*' in (20b) corresponds to '*have to*' in counterfactuals, famously called a "syntactic peculiarity" by Lewis [27]. The parallelism between the local/global distinction in indicatives and the phenomenon of "backtracking" counterfactuals was pointed out in Kaufmann [21], but awaits further analysis. I will not deal with it in this paper.

formally accounted for. This choice deserves some discussion. I will call the two options "semantic" and "pragmatic," respectively.

The semantic option treats the local/global distinction as an ambiguity in the value assignment. The upper lines in each of (16) and (19) can be read as the expectation of a random variable, defined with and without sensitivity to causal dependencies, respectively, and corresponding to the assignments $\mathbb{V}^*$ and V^*. Accordingly, we would derive two evaluation functions $\mathbb{V}$ and V for the local and global interpretation, respectively:

(21)
$$\mathbb{V}(S \to L)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SL)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{L})(\omega) = 1 \\ \mathrm{P}(L|SW) \text{ if } \mathrm{V}(\overline{S}W)(\omega) = 1 \\ \mathrm{P}(L|S\overline{W}) \text{ if } \mathrm{V}(\overline{S}\,\overline{W})(\omega) = 1 \end{cases}$$

(22)
$$\mathrm{V}(S \to L)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SL)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{L})(\omega) = 1 \\ \mathrm{P}(L|S) \text{ if } \mathrm{V}(\overline{S})(\omega) = 1 \end{cases}$$

The pragmatic account, in contrast, assumes that the very same conditional is interpreted with respect to different probability distributions. This approach is suggested by the lower line in each of (16) and (19). I call it "pragmatic" because it appeals to the inference involved in evaluating the conditional, corresponding to different ways of weighing the values assigned according to (21):

(23)
$$\mathbb{P}(S \to L) = \mathrm{P}(L|SW)\mathrm{P}(W) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W})$$

(24)
$$\mathrm{P}(L|S) = \mathrm{P}(L|SW)\mathrm{P}(W|S) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W}|S)$$

Both accounts predict the same probabilities for the conditional, so the choice between them must be motivated by other considerations. Without presuming to settle the issue, I will adopt the pragmatic account. There are good reasons for doing so. One is that the unified treatment of indicative and counterfactual conditionals, which I consider the best justification for the assignment of intermediate values at non-antecedent worlds, does not sit well with the postulation of a semantic ambiguity for indicatives alone. To be sure, the sentence does have both readings, but any corresponding uncertainty as to its value at particular worlds should be treated as uncertainty about the causal relations in those worlds. Moreover, the pragmatic account naturally establishes a straightforward connection between the two readings, which has no analog under the semantic account: The global reading is obtained from the local one by conditioning the probability distribution upon the antecedent. This step, shown in the difference between (23) to (24) above, goes against the flow of causation. Kaufmann [21] suggests that the abductive, non-causal nature of the inference is responsible for its optionality in examples of this kind.

4.3 Back to Complex Conditionals

I leave these considerations behind now and return to complex conditionals. The purpose of discussing the local/global dichotomy at such length was to prepare the ground for a corresponding distinction in the treatment of those complex sentences. The modifications introduced in this section will turn out instrumental in obtaining intuitively correct probabilities.

In particular, the values assigned to the first-order conditionals that form the constituents of more complex sentences will generally be *local*. For the reasons I gave above, I do not assume that conditionals are ambiguous between local and global *values*. They do have local and global *probabilities*, but as a consequence of the decision not to treat this variability as a semantic ambiguity, no local/global distinction is predicted for embedded conditionals. Technically, this is because the global reading comes about by conditionalization on the antecedent. In a probabilistic Stalnaker Bernoulli model, an expression like '$\mathbb{P}^*(D|A \to B)$' is meaningful and defined whenever $\mathbb{P}^*(A \to B)$ is positive. However, just as in the standard Bayesian calculus, conditionalization is only defined for top-level conditionals. Thus while we can speak meaningfully of '$\mathbb{P}^*(D|A \to B)$', expressions like '$\mathbb{P}^*(D|(B|A))$' are meaningless.

Before entering the next section, I should briefly point out what this implies for the four classes of sentences introduced in Section 3. Conjunctions of conditionals will not exhibit the local/global distinction, a prediction for which I have not as yet found any counterexamples. For conditionals with conditional antecedents, the distinction would in principle be available; however, recall that the representation of causality in the model (Section 4.1 above) bars conditionals from entering into causal relations. As a result, Definition 12 does not give a distinct local interpretation for conditionals with conditional antecedents. They have only global readings for that reason. It is only for conditionals with non-conditional antecedents that a local reading is predicted which may differ from the global one. Below, I will discuss examples in which this difference is illustrated.

5 An Analysis of Some Examples

In this section I will argue, based on a number of examples, that the new assignment introduced in Section 4 is a significant improvement over the earlier version. The argument will go as follows: A sentence of the relevant form is discussed in the context of some scenario characterized by an assignment of probabilities to certain atomic and complex sentences. The formulas from Section 3 yield a probability for the sentence in question, which however differs more or less dramatically from intuitions given the scenario. I will then show that the causally sensitive value assignment yields intuitively plausible values, provided that certain causal relations hold in the scenario.

Such assumptions about causality are usually not explicitly given in the literature. Thus what the arguments will establish is that *there is* a causal extension of the model under which the sentence receives a sensible probability. That does not prove that this causal structure is the "true" one, or the one that its original proponent had in mind, although in each case there will be good evidence that it is, furnished by both the improved predictions about the sentence in question and the close affinity between the revised values and intuitions about counterfactuals.

Some calculations are unavoidable in this section, but most details will be relegated to footnotes. In the interest of brevity, I will not discuss the consequences of the new definition for the assignment $\mathbb{V}$ of values at worlds in Ω. This assignment can be inferred from $\mathbb{V}^*$ and $\Pr^*$ along the lines of Section 3.

5.1 Harry: The Scenario

I will discuss all four classes of complex conditionals with respect to one scenario, which I introduce in this subsection. In addition, I will also consider examples which have been discussed elsewhere in the literature. We will see that those additional examples have some peculiarities which simplify the arguments based on them, but which also limit the generality of those arguments somewhat. The scenario I present here does not suffer from those limitations.

Suppose our friend Harry is applying simultaneously for a job and for participation in a TV quiz show. The two applications are independent. If he is selected for participation in the show (S), he may or may not win (W). The job application involves taking a test (T), which he may or may not pass (P). The four sentences of interest are listed below.

(25) a. If Harry takes the test, he will pass, and if he is selected for the show, he will win.

$$(T \to P) \wedge (S \to W)$$

b. If Harry will pass the test if he takes it, he will win in the show if he is selected.

$$(T \to P) \to (S \to W)$$

c. If Harry passes the test, he will win if he is selected for show.

$$P \to (S \to W)$$

d. If Harry will win if he is selected for the show, he will pass the test.

$$(S \to W) \to P$$

Furthermore, there is an unmentioned but relevant background factor $V(I)$, indicating whether or not Harry is intelligent. I assume for simplicity that $V(I)$ is binary like the other sentences, thus intelligence is not measured in degrees. The probabilities are given in (26).

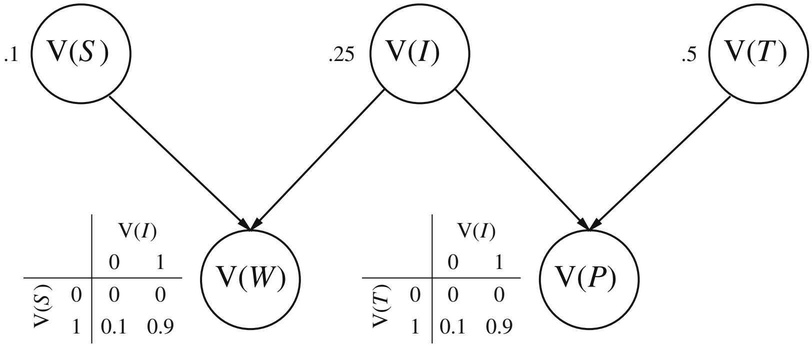

Fig. 1 Independence assumptions and probabilities in (26)

(26)	Harry is selected for the show	$P(S) = 0.1$
	Harry is intelligent	$P(I) = 0.25$
	Harry takes the test	$P(T) = 0.5$
	Harry wins in the show	$P(W \mid SI) = 0.9$ $P(W \mid S\overline{I}) = 0.1$
	Harry passes the test	$P(P \mid TI) = 0.9$ $P(P \mid T\overline{I}) = 0.1$

From these numbers it follows that $P(P) = 0.15$ and $P(W) = 0.03$.[13] In addition, I make some assumptions about causal independencies. These are displayed in the graph in Fig. 1. Following the conventions in some applications of Bayesian networks, the arrows are intended to follow the flow of causation: Whether Harry wins in the TV show does not only depend on whether he is selected, but also on whether he is intelligent. Similarly for the test: His chances of passing depend on his intelligence. No causal dependence holds in the opposite direction. Thus while his performance on the test may provide *evidence* for or against his being intelligent, it does not *make* him more or less so.

Finally, the variables in the upper row are assumed to be causally independent of each other. I make this assumption not so much because it simplifies the calculations (although it does), but because it seems reasonable in this scenario. Whether Harry is selected for the show is independent of his intelligence. The lack of an arrow between these variables in either direction implies that they are also *stochastically* independent as long as $V(W)$ is not observed. As a consequence, the two readings (local and global) of the condi-

[13] $P(P) = P(P|TI)P(TI) + P(P|T\overline{I})P(T\overline{I})$
$= P(P|TI)P(T)P(I) + P(P|T\overline{I})P(T)P(\overline{I})$ [independence of $V(T)$ and $V(I)$]
$= 0.9 \times 0.25 \times 0.5 + 0.1 \times 0.75 \times 0.5 = 0.15$
$P(W) = 0.9 \times 0.1 \times 0.25 + 0.1 \times 0.1 \times 0.75 = 0.03$ by similar reasoning.

tional *'If Harry is selected, he will win'* are equivalent in this scenario, since conditionalization upon the antecedent does not affect any other variables. The same holds for $\mathrm{V}(T)$ and $\mathrm{V}(P)$ on the other side of the graph. Finally, from the numbers in (26) it follows that both of these probabilities are 0.3.[14]

Notice also that Harry's chances of winning in the show (if selected) and his performance on the test (if he takes it) are conditionally independent, given I. Thus for instance, even though $\mathrm{P}(W|SP)$ is not equal to $\mathrm{P}(W|S)$, $\mathrm{P}(W|SPI)$ equals $\mathrm{P}(W|SI)$. Facts of this sort will help simplify some of the calculations below.

5.2 Counterfactuals

Before turning to the evaluation of complex conditionals in this scenario, we should confirm that the values indicative conditionals receive at non-antecedent worlds match intuitions about their counterfactual counterparts.

(27) a. If Harry had been selected for the show, he would have won.
b. If Harry is selected for the show, he will win.
c. $S \to W$

Recall that I assume that the values assigned to (27c), given in (28), are those of (27a) as well as (27b):

(28)
$$\begin{aligned}\mathbb{V}^*(S \to W)(\omega^*) = \mathrm{V}^*(W)(\omega^*(n))&\\ \text{for the least } n \text{ such that } \mathrm{V}^*(S)(\omega^*(n)) = 1,&\\ \mathrm{V}^*(I)(\omega^*(n)) = \mathrm{V}^*(I)(\omega^*),\ \mathrm{V}^*(T)(\omega^*(n)) = \mathrm{V}^*(T)(\omega^*),&\\ \text{and } \mathrm{V}^*(P)(\omega^*(n)) = \mathrm{V}^*(P)(\omega^*)&\end{aligned}$$

It is easy to see that these values at sequences in Ω^* correspond to those in (29) at worlds in Ω:

(29)
$$\mathbb{V}(S \to W)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SW)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{W})(\omega) = 1 \\ \mathrm{P}(W|SITP) \text{ if } \mathrm{V}(\overline{S}ITP)(\omega) = 1 \\ \dots \\ \mathrm{P}(W|S\overline{I}\,\overline{T}\,\overline{P}) \text{ if } \mathrm{V}(\overline{S}\,\overline{I}\,\overline{T}\,\overline{P})(\omega) = 1 \end{cases}$$

Furthermore, due to the conditional independence between $\{S, W\}$ and $\{T, P\}$, given I, we know that $\mathrm{P}(W|SITP) = \mathrm{P}(W|SI)$ etc., so (29) is equivalent to the simpler (30).

[14]
$$\begin{aligned}\mathrm{P}(W|S) &= \mathrm{P}(W|SI)\mathrm{P}(I|S) + \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{I}|S)\\ &= \mathrm{P}(W|SI)\mathrm{P}(I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{I})\\ &= \mathrm{P}(S \to W) = 0.9 \times 0.25 + 0.1 \times 0.75 = 0.3\end{aligned}$$
$\mathrm{P}(P|T) = \mathrm{P}(T \to P) = 0.9 \times 0.25 + 0.1 \times 0.75 = 0.3$ by similar reasoning.

(30) $$\mathbb{V}(S \to W)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SW)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{W})(\omega) = 1 \\ 0.9 = \mathrm{P}(W|SI) \text{ if } \mathrm{V}(\overline{S}I)(\omega) = 1 \\ 0.1 = \mathrm{P}(W|S\overline{I}) \text{ if } \mathrm{V}(\overline{S}\,\overline{I})(\omega) = 1 \end{cases}$$

I take it to be self-evident that the values in (30), interpreted as those of the counterfactual, are more in line with intuitions than the ones under the original definition, which would assign the conditional probability $\mathrm{P}(W|S) = 0.3$ at all $\overline{S}$-worlds (i.e., those at which Harry is not selected for the show), regardless of whether Harry is intelligent in those worlds or not. Intuitively, the value of the counterfactual does indeed depend on Harry's intelligence in just the way it does in (30).

This example does not illustrate the local/global distinction for indicative conditionals. The local probability of the indicative conditional (27) under this assignment is the same as the corresponding conditional probability, as shown in Footnote 14. For a case in which the two differ, consider (31a) and (31b).

(31) a. If Harry had passed the test, he would have won in the show.
b. If Harry passes the test, he will win in the show.
c. $P \to W$

Since all the relevant variables are non-descendants of $V(P)$ in the causal structure, the assignment of values to this sentence is as in (32), making the conditional equivalent to its consequent:

(32) $$\mathbb{V}^*(P \to W)(\omega^*) = \mathrm{V}^*(W)(\omega^*(n))$$
$$\text{for the least } n \text{ such that } \mathrm{V}(P)(\omega^*(n)) = 1,$$
$$\mathrm{V}^*(T)(\omega^*(n)) = \mathrm{V}^*(T)(\omega^*), \mathrm{V}^*(I)(\omega^*(n)) = \mathrm{V}^*(I)(\omega^*),$$
$$\mathrm{V}^*(S)(\omega^*(n)) = \mathrm{V}^*(S)(\omega^*), \mathrm{V}^*(W)(\omega^*(n)) = \mathrm{V}^*(W)(\omega^*)$$

Clearly, since the value of W is among those that are held constant in (32), the "unstarred" value of the conditional at worlds in Ω is just that of $\mathrm{V}(W)$:

(33) $$\mathrm{V}(P \to W)(\omega) = \mathrm{V}(W)(\omega)$$

The reading that is captured by this assignment is best paraphrased as in (34). This reflects the assumption that Harry's performance on the test has no causal effect on his selection for the quiz or his performance therein.

(34) (Even) if Harry had passed the test, he would (still) have won (or not have won, as the case may be at ω).

What this predicts about the indicative conditional in (31b) may appear less intuitive at first. The probability of the conditional under this interpretation is just the unconditional probability that Harry will win, which is 0.03, as was shown in Footnote 13. This may seem odd because intuitively, Harry's passing the test provides evidence that he is intelligent, hence that his prospects for the TV quiz are better than we thought they would be. This intuition is not reflected in (32). Rather, just like for the counterfactual, the best paraphrase of the indicative is concessive:

(35) (Even) if Harry passes the test, he will (still) not win in the show (since I think it unlikely that he is intelligent, and his passing the test does not make him so).

The interpretation that accords better with intuitions in this case corresponds to the global reading of the conditional, which incorporates the "backtracking" inference from Harry's passing the test to his being intelligent. The global probability is just the conditional probability of W, given P, which is 0.07.[15] Now, this may still appear to be rather low. However, recall that the probability that Harry wins cannot possibly exceed the probability that he is selected for the show, and the latter is just 0.1 and independent of his passing the test. Thus 0.07 is a good reflection of intuitions after all.

5.3 Conjoined Conditionals

Moving on to more complex sentences, consider first the case of conjunctions of conditionals, such as (36).

(36) a. If Harry takes the test, he will pass, and if he is selected for the show, he will win.
b. $(T \to P) \wedge (S \to W)$

Recall that under the original assignment, the probability of this sentence was the following:

(37)
$$\mathrm{P}^*((T \to P) \wedge (S \to W)) = \frac{\mathrm{P}(TPSW) + \mathrm{P}(W|S)P(TP\overline{S}) + \mathrm{P}(P|T)\mathrm{P}(SW\overline{T})}{\mathrm{P}(T \vee S)}$$

Using the numbers given in the scenario, we can determine the probability assigned to (36) under (37). Intuitively, both of the conjuncts in (36) are highly correlated with Harry's being intelligent, so the probability of the conjunction

[15] $\mathrm{P}(I|P) = \mathrm{P}(P|I)\mathrm{P}(I)/[\mathrm{P}(P|I)\mathrm{P}(I) + \mathrm{P}(P|\overline{I})\mathrm{P}(\overline{I})]$
$= 0.9 \times 0.25/[0.9 \times 0.25 + 0.1 \times 0.75] = 0.225/0.3 = 0.75$

Next, recall that $\mathrm{P}(W\overline{S}) = \mathrm{P}(P\overline{T}) = 0$. Thus

$$\begin{aligned}\mathrm{P}(W|P) &= \mathrm{P}(W|SITP)\mathrm{P}(S|ITP)\mathrm{P}(I|TP)\mathrm{P}(T|P)\\ &\quad + \mathrm{P}(W|S\overline{I}TP)\mathrm{P}(S|\overline{I}TP)\mathrm{P}(\overline{I}|TP)\mathrm{P}(T|P)\\ &= \mathrm{P}(W|SI)\mathrm{P}(S)\mathrm{P}(I|P) + \mathrm{P}(W|S\overline{I})\mathrm{P}(S)\mathrm{P}(\overline{I}|P)\\ &\quad \text{[due to independence assumptions and since } \mathrm{P}(T|P) = 1]\\ &= 0.9 \times 0.1 \times 0.75 + 0.1 \times 0.1 \times 0.25 = 0.07\end{aligned}$$

should be only slightly less than 0.25, the probability that he is intelligent. However, according to (37) the probability is only about 0.1.[16]

What does the new assignment predict about this sentence? Recall how, given the causal dependencies in the scenario, the values are defined:

(38) $$\mathbb{V}^*((T \to P) \wedge (S \to W))(\omega^*)$$
$$= \mathbb{V}^*(T \to P)(\omega^*) \times \mathbb{V}^*(S \to W)(\omega^*)$$
$$= \begin{cases} 1 \text{ if } \mathrm{V}^*(P)(\omega^*(n)) = 1 \text{ for the least } n \text{ such that} \\ \quad \mathrm{V}^*(T)(\omega^*(n)) = 1, \mathrm{V}^*(S)(\omega^*(n)) = \mathrm{V}^*(S)(\omega^*), \\ \quad \mathrm{V}^*(W)(\omega^*(n)) = \mathrm{V}^*(W)(\omega^*), \mathrm{V}^*(I)(\omega^*(n)) = \mathrm{V}^*(I)(\omega^*) \\ \quad \text{and} \\ \quad \mathrm{V}^*(W)(\omega^*(m)) = 1 \text{ for the least } m \text{ such that} \\ \quad \mathrm{V}^*(S)(\omega^*(m)) = 1, \mathrm{V}^*(T)(\omega^*(m)) = \mathrm{V}^*(T)(\omega^*), \\ \quad \mathrm{V}^*(P)(\omega^*(m)) = \mathrm{V}^*(P)(\omega^*), \mathrm{V}^*(I)(\omega^*(m)) = \mathrm{V}^*(I)(\omega^*) \\ 0 \text{ otherwise} \end{cases}$$

In calculating the value of the left conjunct at a given world sequence ω^* at whose first world the antecedent T is false, only those worlds in the sequence are relevant which agree with the first world on the values of all relevant variables except for the consequent: S, W, and I. Likewise, in calculating the second conjunct, the values of T, P and I are held constant.

As before, the conjunction is true whenever all four atomic constituents are true at the first world in the sequence (39). The simplification in the last line of (40) comes about due to the conditional independencies inherent in this scenario.

(39) $$\mathbb{P}^*(((T \to P) \wedge (S \to W)) \wedge TPSW) = \mathrm{P}(TPSW)$$

16
$$\begin{aligned} \mathrm{P}(TPSW) &= \mathrm{P}(T)\mathrm{P}(S|T)[\mathrm{P}(I|TS)\mathrm{P}(W|ITS)\mathrm{P}(P|WITS) \\ &\quad + \mathrm{P}(\overline{I}|TS)\mathrm{P}(W|\overline{I}TS)\mathrm{P}(P|W\overline{I}TS)] \\ &= \mathrm{P}(T)\mathrm{P}(S)[\mathrm{P}(I)\mathrm{P}(W|IS)\mathrm{P}(P|IT) + \mathrm{P}(\overline{I})\mathrm{P}(W\overline{I}S)\mathrm{P}(P|\overline{I}T)] \\ &= 0.5 \times 0.1 \times [0.25 \times 0.9 \times 0.9 + 0.75 \times 0.1 \times 0.1] = 0.0105 \\ \mathrm{P}(TP\overline{S}) &= \mathrm{P}(T)\mathrm{P}(P|T)\mathrm{P}(\overline{S}) = 0.5 \times 0.3 \times 0.9 = 0.135 \\ \mathrm{P}(SW\overline{T}) &= \mathrm{P}(S)\mathrm{P}(W|S)\mathrm{P}(\overline{T}) = 0.1 \times 0.3 \times 0.5 = 0.015 \\ \mathrm{P}(T \vee S) &= \mathrm{P}(T) + \mathrm{P}(S) - \mathrm{P}(T)\mathrm{P}(S) = 0.5 + 0.1 - 0.05 = 0.55 \\ \mathrm{P}(36\mathrm{b}) &= [0.0105 + 0.3 \times 0.135 + 0.3 \times 0.015]/.55 = 0.0555/.55 \approx 0.1009 \end{aligned}$$

(40)
$$\begin{aligned}&\mathbb{P}^*(((T \to P) \wedge (S \to W)) \wedge TP\overline{S}I)\\&= \mathrm{P}(TP\overline{S}I)\mathrm{P}(TPSIW) + \mathrm{P}(TP\overline{S}I)\mathrm{P}(\overline{TPSI})\mathrm{P}(TPSIW)\\&\quad + \mathrm{P}(TP\overline{S}I)\mathrm{P}(\overline{TPSI})^2\mathrm{P}(TPSIW)\ldots\\&= \mathrm{P}(W|TPSI)\mathrm{P}(TP\overline{S}I) = \mathrm{P}(W|SI)\mathrm{P}(TP\overline{S}I)\end{aligned}$$

The other cases in which one of the conjuncts is true and the other has a false antecedent are similar to (40). Overall, then, the probability of the conditional under the causal value assignment is as follows:[17]

(41)
$$\begin{aligned}&\mathbb{P}^*((T \to P) \wedge (S \to W))\\&= \frac{1}{\mathrm{P}(T \vee S)}\begin{bmatrix}\mathrm{P}(TPSW)\\+\mathrm{P}(W|SI)\mathrm{P}(TP\overline{S}I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(TP\overline{S}\,\overline{I})\\+\mathrm{P}(P|TI)\mathrm{P}(SW\overline{T}I) + \mathrm{P}(P|T\overline{I})\mathrm{P}(SW\overline{T}\,\overline{I})\end{bmatrix}\\&= 0.21\end{aligned}$$

This result accords much better with intuitions. Moreover, the values assigned to the conjuncts at worlds at which their antecedents are false are also plausible with regard to the corresponding counterfactuals.

The main difference between (37) and (41) consists in the role of $\mathrm{V}^*(I)$, the indicator of Harry's intelligence. In general, each of the two conditional conjuncts in a sentence like (36) may leave different causally relevant background variables unaffected, but in the present case, the independence assumption ensured that this did not make a difference in the end result. Nor have such cases been discussed in the literature. In fact, the literature is limited to even simpler examples. I now turn to one such example, one which involves not only the same independent background variable for both conjuncts, but also some additional peculiarities. Nevertheless, we will see that by spelling out certain implicit assumptions about causal dependencies, we obtain a better analysis of the example than was hitherto available.

17
$$\begin{aligned}\mathrm{P}(TP\overline{S}I) &= \mathrm{P}(T)\mathrm{P}(P|T)\mathrm{P}(\overline{S}|TP)\mathrm{P}(I|\overline{S}TP)\\&= \mathrm{P}(T)\mathrm{P}(P|T)\mathrm{P}(\overline{S})\mathrm{P}(I|TP) = 0.5 \times 0.3 \times 0.9 \times 0.75 = 0.10125\\\mathrm{P}(TP\overline{S}\,\overline{I}) &= \mathrm{P}(T)\mathrm{P}(P|T)\mathrm{P}(\overline{S})\mathrm{P}(\overline{I}|TP) = 0.5 \times 0.3 \times 0.9 \times 0.25 = 0.03375\\\mathrm{P}(SW\overline{T}I) &= \mathrm{P}(S)\mathrm{P}(W|S)\mathrm{P}(\overline{T})\mathrm{P}(I|SW) = 0.1 \times 0.3 \times 0.5 \times 0.75 = 0.01125\\\mathrm{P}(SW\overline{T}\,\overline{I}) &= \mathrm{P}(S)\mathrm{P}(W|S)\mathrm{P}(\overline{T})\mathrm{P}(\overline{I}|SW) = 0.1 \times 0.3 \times 0.5 \times 0.25 = 0.00375\\\mathrm{P}(41) &= \frac{0.0105 + 0.9 \times 0.10125 + 0.1 \times 0.03375 + 0.9 \times 0.01125 + 0.1 \times 0.00375}{0.55}\\&= 0.1155/0.55 = 0.21\end{aligned}$$

5.3.1 The Werewolf

Lance [25] used the following example to show that the formula for conjunctions of conditionals yields counterintuitive results.[18] A werewolf lives nearby who under full moon, such as tonight, always stalks one of two neighborhoods, *n* or *m*, completely at random. Anybody who goes outside in the neighborhood stalked by the werewolf is killed with absolute certainty. We are in a house in neighborhood *n*. Suddenly we notice that Jones, who was in the house earlier, has not been seen for a while. There is a fifty percent chance that he left the house, but perhaps he is upstairs. There are two doors, *a* and *c*, through which he might have left with equal probability. We have no reason to think that Jones may have died for any reason but the werewolf. To summarize, the following are the probabilities:

(42)	N	The werewolf is stalking *n*	$P(N) = 0.5$
	$\overline{N}$	The werewolf is stalking *m*	$P(\overline{N}) = 0.5$
	S	Jones is inside	$P(S) = 0.5$
	A	Jones left through door *a*	$P(A) = 0.25$
	C	Jones left through door *c*	$P(C) = 0.25$
	K	Jones is killed	$P(K \mid (A \vee C) \wedge N) = 1$

Now consider the sentence in (43):

(43) a. If Jones left through door *a* he was killed, and if he left through door *c* he was killed.
b. $(A \to K) \wedge (C \to K)$

This sentence has two peculiar properties. First, the consequents of both constituent conditionals are the same. Secondly, and more importantly, the scenario is set up in such a way that the antecedents A and C of the constituents are mutually incompatible, hence the probability that both A and C are true is 0. As was first pointed out by Edgington [8], in this case the right-hand side of (37) simplifies rather dramatically to $P(K|C)P(K|A)$.[19] Now it is implied by the scenario that each of these conditional probabilities is 0.5,[20]

[18] Lance's paper was a response to McGee [30], who had proposed the same formula for conjoined conditionals.

[19] In general, if $P(AC) = 0$, then $P(ABCD) = 0$, $P(AB\overline{C}) = P(AB)$, $P(\overline{A}CD) = P(CD)$, and $P(A \vee C) = P(A) + P(C)$; thus:

$$\begin{aligned} P((A \to B) \wedge (C \to D)) &= [P(D|C)P(AB) + P(B|A)P(CD)]/[P(A) + P(C)] \\ &= P(D|C)P(B|A)[P(A) + P(C)]/[P(A) + P(C)] \\ &= P(D|C)P(B|A) \end{aligned}$$

[20] $$\begin{aligned} P(K|A) &= P(KA)/P(A) = [P(KAN) + P(KA\overline{N})]/P(A) \\ &= P(AN)/P(A) \text{ [since } P(KAN) = P(AN) \text{ and } P(KA\overline{N}) = 0] \\ &= P(N|A) = P(N) \text{ [since } A \text{ and } N \text{ are independent]} \end{aligned}$$

The case of $P(K|C)$ is parallel.

and so we arrive at the prediction that $P((A \to K) \wedge (C \to K)) = 0.25$. But, Lance argues convincingly, this is too low. Intuitively, the whole conjunction is certainly true if the werewolf is stalking n, and certainly false if she is stalking m. So its truth is fully determined by whether she is stalking our neighborhood, and one would expect a probability of 0.5 instead of 0.25.

To see where the source of the problem lies, we should consider the value assignment in its unsimplified form. Under the original, non-causal assignment, the intermediate value assigned to $A \to K$ at those worlds in which Jones did not leave through door a is $P(K|A)$; similarly, it is $P(C \to K)$ at worlds in which he did not leave through door c. The probability that he left through door c and was killed is $P(CK) = P(\overline{A}CK) = 0.125$; similarly for $P(AK)$. Thus according to (37):

(44)
$$\begin{aligned} &P^*((A \to K) \wedge (C \to K)) \\ &= \frac{P(ACK) + P(K|A)P(\overline{A}CK) + P(K|C)P(AK\overline{C})}{P(A \vee C)} \\ &= \frac{0 + 0.5 \times 0.125 + 0.5 \times 0.125}{0.5} = 0.25 \end{aligned}$$

Now consider the worlds in $\overline{A}CK$, where Jones left through door c and was killed. The value assigned to (45a) at those worlds is $P(K|A) = 0.5$.

(45) a. If Jones left through door a, he was killed.
b. If Jones had left through door a, he would have been killed.

What are the facts at those worlds? Jones leaves through door c and is killed; this can only happen if the werewolf is stalking neighborhood n. So all worlds in $\overline{A}CK$ are N-worlds. Suppose Jones' behavior does not affect the werewolf's: Regardless of whether Jones leaves through door a or c or stays inside, the werewolf still stalks whichever neighborhood she stalks. (It is reasonable to make this assumption, although Lance does not make it explicit. The reader may care to verify that intuitions about the conditional would be different without it.) Likewise, at the worlds where Jones left through door c and is *not* killed, the werewolf must be stalking m. But then, if Jones had left through door a, he would not have been killed either. Based on these intuitions about causal independencies and the counterfactual in (45b), it would seem that at those worlds where Jones left through door c and is killed, the value of (45a) should be $P(K|AN) = 1$, and where he leaves through door c and is not killed, the value should be $P(K|AM) = 0$. Instead, the value is uniformly $P(K|A) = 0.5$.

The situation is mirrored at those worlds in which Jones left through door a. Where he is killed, he would certainly have been killed if he had left through door c. Where he is not killed, he would not have been killed if he had left through door c, either.

Suppose $V^*(N)$, the variable indicating whether the werewolf is stalking neighborhood n, is causally independent from $V^*(A)$, $V^*(C)$ and $V^*(S)$. Then the two conjuncts of (43a) are interpreted as equivalent to (46a) at N-worlds, and to (46b) at $\overline{N}$-worlds.

(46) a. If Jones left through door *a*/*c* and the werewolf is stalking *n*, Jones was killed.
b. If Jones left through door *a*/*c* and the werewolf is stalking *m*, Jones was killed.

Thus instead of (44) above, we now have (47), which simplifies to (*) due to the fact that in the scenario, $\mathrm{P}(ACK) = \mathrm{P}(K\overline{N}) = 0$.

(47)
$$\mathbb{P}^*((A \to K) \wedge (C \to K))$$
$$= \frac{1}{\mathrm{P}(A \vee C)} \begin{bmatrix} \mathrm{P}(ACK) \\ +\mathrm{P}(K|AN)\mathrm{P}(\overline{A}CKN) + \mathrm{P}(K|CN)\mathrm{P}(AK\overline{C}N) \\ +\mathrm{P}(K|A\overline{N})\mathrm{P}(\overline{A}CK\overline{N}) + \mathrm{P}(K|C\overline{N})\mathrm{P}(AK\overline{C}\,\overline{N}) \end{bmatrix}$$

(*)
$$= \frac{\mathrm{P}(K|AN)\mathrm{P}(\overline{A}CKN) + \mathrm{P}(K|CN)\mathrm{P}(AK\overline{C}N)}{\mathrm{P}(A \vee C)}$$
$$= \frac{1 \times 0.125 + 1 \times 0.125}{0.5} = 0.5$$

This is the probability that Lance would deem correct. Further advantages of the causally sensitive interpretation can be observed with respect to a variation on Lance's original sentence. Consider (48).

(48) a. If Jones left through door *a* he was killed, and if he left through door *c* he was *not* killed.
b. $(A \to K) \wedge (C \to \overline{K})$

Intuitively, the probability of (48) should be 0. However, the original formula (37) predicts otherwise:

(49)
$$\mathrm{P}^*((A \to K) \wedge (A \to \overline{K}))$$
$$= \frac{\mathrm{P}(K|A)\mathrm{P}(\overline{A}C\overline{K}) + \mathrm{P}(\overline{K}|C)\mathrm{P}(AK\overline{C})}{\mathrm{P}(A \vee C)}$$
$$= \frac{0 + 0.5 \times 0.125 + 0.5 \times 0.125}{0.5} = 0.25$$

The problem is the same as before: At worlds in which Jones left through door *c* and was not killed, he would not have been killed if he had left through door *a*, for these are $\overline{N}$-worlds and remain so regardless of Jones's whereabouts. An assignment that is sensitive, in the above sense, to this fact yields the correct probability:

(50) $$\begin{aligned}\mathbb{P}^*((A \to K) \wedge (C \to \overline{K})) &= \frac{\mathrm{P}(K|A\overline{N})\mathrm{P}(\overline{A}C\overline{K}\,\overline{N}) + \mathrm{P}(\overline{K}|CN)\mathrm{P}(AK\overline{C}N)}{\mathrm{P}(A \vee C)}\\ &= \frac{0 \times 0.125 + 0 \times 0.125}{0.5} = 0\end{aligned}$$

Thus the new value assignment handles objections like Lance's quite well.[21]

5.4 Conditional Conditionals

Following the order in Section 3, I turn next to conditionals with conditional antecedents and consequents. In the "Harry" scenario, the sentence to consider is (51):

(51) a. If Harry will pass if he takes the test, he will win if he is selected for the show.
b. $(T \to P) \to (S \to W)$

In the previous section on conjunctions of conditionals, we saw that the original value assignment leads to poor predictions because it fails to capture the fact that the two constituents $T \to P$ and $S \to W$ are closely correlated. In the present case, this fact gives rise to a similar problem. Intuitively, the probability of (51) should be high in virtue of this correlation: The antecedent provides evidence for Harry's being intelligent, which in turn makes the con-

[21] Another example is offered by McDermott [29], who, in discussing McGee's account, considers the sentence in (i) in the context of the impending toss of a fair die.

(i) If it's odd it will be below three, and if it's even it will be above three.

The probability assigned to this sentence under the original account is as follows (writing 'O' and 'E' for *odd* and *even* and 'B' and 'A' for *below 3* and *above 3*, respectively):

$$\begin{aligned}&\mathrm{P}((O \to B) \wedge (E \to A))\\ &\quad = [\mathrm{P}(OBEA) + \mathrm{P}(B|O)\mathrm{P}(EA\overline{O}) + \mathrm{P}(A|E)\mathrm{P}(OB\overline{E})]/\mathrm{P}(O \vee E)\\ &\quad = [0 + 1/3 \times 2/6 + 2/3 \times 1/6]/1 = 2/9\end{aligned}$$

McDermott proclaims that this probability is much too low for the example, "about which everyone's intuitions are clear" (p. 26). McDermott's intuition is that the sentence "is true if the result is a six, four, or one, false otherwise; so its assertability [as measured by the probability - SK] is 1/2." I cannot comment on this any further because I simply do not share that intuition. In my mind, (i) describes a highly unlikely state of affairs, certainly less likely than, say, the die's coming up odd.

sequent likely. In fact, however, the probability predicted under the original assignment is rather low:[22]

(52) $$\mathrm{P}^*((T \to P) \to (S \to W)) = \frac{\mathrm{P}(TPSW) + \mathrm{P}(W|S)\mathrm{P}(TP\overline{S}) + \mathrm{P}(P|T)\mathrm{P}(\overline{T}SW)}{\mathrm{P}(T \vee S)\mathrm{P}(P|T)} \approx 0.336$$

What does the revised assignment predict in this case? Recall that in Section 3, the probability for these sentences came out equivalent to $\mathrm{P}^*((T \to P) \wedge (S \to W))/\mathrm{P}^*(T \to P)$. Now we only need to replace the new values for the numerator and denominator of this expression. The former was derived in the last section; the latter is 0.3 (cf. Footnote 14). Thus:

(53) $$\mathbb{P}^*((T \to P) \to (S \to W)) = 0.21/0.3 = 0.7$$

As before, the causally sensitive value assignment improves the predictions of the account. Recall that while the conditionals in the antecedent and consequent of this sentence now receive their "local" interpretation, there is no corresponding local/global distinction with regard to the main conditional connective in the sentence, since it has a conditional antecedent.

5.4.1 The Werewolf

Another variant of Lance's example is illuminating in this context as well:[23]

(54) a. If Jones was killed if he left through door *a*, he was killed if he left through door *c*.
b. $(A \to K) \to (C \to K)$

Under the original assignment, the probability of (54) is $\mathrm{P}(K|C) = 0.5$.[24] But this is too low: The sentence is highly likely in Lance's scenario. As before, the intuitively correct probability is predicted if the values of conditionals with

[22] $\mathrm{P}^*((T \to P) \to (S \to W)) = \mathrm{P}^*((T \to P) \wedge (S \to W))/\mathrm{P}^*(P|T) = 0.1009/0.3 \approx 0.3363$

[23] An anonymous reviewer of an earlier version of this paper complained about a variant of this sentence (whose consequent was introduced by '*then*'), stating that the conditional is barely intelligible. Other informants do find the sentence acceptable, if somewhat unusual, and have fairly clear intuitions about it. My suspicion is that the past reference in the antecedent (which makes the conditional an "epistemic" one) causes the confusion by some as-yet ill-understood mechanism. The concerned reader may prefer to substitute the form in (i) for (54).

(i) If Jones will be killed if he leaves through door *a*, he will be killed if he leaves through door *c*.

[24] Recall from Footnote 7 that due to the peculiarities of the scenario, the probability of the conjunction is $\mathrm{P}(K|A)\mathrm{P}(K|C)$. Thus the probability of (54) is $\mathrm{P}(K|A)\mathrm{P}(K|C)/\mathrm{P}(K|A)$.

false antecedents are made sensitive to the werewolf's whereabouts. Under the revised value assignment, we get the desired result:

(55)
$$\begin{aligned}&\mathbb{P}^*((A \to K) \to (C \to K))\\ &= \frac{P(ACK) + P(K|AN)P(\overline{A}KC) + P(K|CN)P(AK\overline{C})}{P(A \vee C)P(K|A)}\\ &= \frac{0 + 1 \times 0.125 + 1 \times 0.125}{0.5 \times 0.5} = \frac{0.25}{0.25} = 1\end{aligned}$$

5.4.2 The Coin

Yet another example of a similar kind, cited by Edgington [8], is the following.

> [A] coin has been selected which is equally likely either double-headed or double-tailed, and is to be tossed soon. 'If it will land heads if first tossed at t, then it will land heads if first tossed at t' [$\neq t$]', seems to me to deserve probability 1, not 1/2. (p. 202)

This example corresponds to Lance's in crucial respects and exhibits the same peculiarities: The antecedents of the embedded conditionals are mutually exclusive, and their consequents are the same. The solution is similar to that in the werewolf example.

5.5 Right-nested Conditionals

Consider next the case of right-nested conditionals. The example (25c) is repeated in (56).

(56) a. If Harry passes the test, he will win if he is selected for the show.
b. $P \to (S \to W)$

Right-nested conditionals are special in that among all the compound sentences I am considering here, they alone have both local and global readings under the revised interpretation. This is because their antecedents do not contain conditionals and can therefore enter into causal relations.

First recall that according to the original formula, the probability of (56) is the following:

(57)
$$P^*(P \to (S \to W)) = P(SW|P) + P(W|S)P(\overline{S}|P)$$

We know from the scenario that $\mathrm{P}(PSW) = 0.0105$,[25] and so $\mathrm{P}(SW|P) = 0.0105/.15 = 0.07$. Since $V^*(S)$ and $V^*(P)$ are independent, (57) becomes (58), a value that is again rather low:

(58)
$$\begin{aligned}\mathrm{P}^*(P \rightarrow (S \rightarrow W)) =& \mathrm{P}(SW|P) + \mathrm{P}(W|S)\mathrm{P}(\overline{S})\\ =& 0.07 + 0.3 \times 0.9 = 0.34\end{aligned}$$

To see what the causally sensitive alternative predicts, I will spell out the values at individual world sequences in Ω^* in some detail. Given the causal structure, this assignment is as follows:

(59)
$$\begin{aligned}&\mathbb{V}^*(P \rightarrow (S \rightarrow W))(\omega^*) = \mathbb{V}^*(S \rightarrow W)(\omega^*(n))\\ &\quad \text{for the least } n \text{ such that } \mathrm{V}^*(P)(\omega^*(n)) = 1,\\ &\quad \mathrm{V}^*(T)(\omega^*(n)) = \mathrm{V}^*(T)(\omega^*), \mathrm{V}^*(I)(\omega^*(n)) = \mathrm{V}^*(I)(\omega^*),\\ &\quad \mathrm{V}(S)(\omega^*(n)) = \mathrm{V}(S)(\omega^*), \mathrm{V}^*(W)(\omega^*(n)) = \mathrm{V}^*(W)(\omega^*)\end{aligned}$$

It is obvious that the whole formula is equivalent to $\mathrm{V}^*(W)$ at all world sequences in which S is true, i.e., Harry is selected for the show, regardless of the values of the other variables. This is because whenever the consequent is not evaluated at ω^*, but at some P-subsequence, that subsequence must agree with ω^* on the values of both $\mathrm{V}^*(S)$ and $\mathrm{V}^*(W)$, and where S is true, the consequent $S \rightarrow W$ is equivalent to W. Thus:

(60)
$$\mathbb{P}^*((P \rightarrow (S \rightarrow W)) \wedge SW) = \mathrm{P}(SW)$$

(61)
$$\mathbb{P}^*((P \rightarrow (S \rightarrow W)) \wedge S\overline{W}) = 0$$

The only sequences that are not covered by this are ones at which S is false. Among them, consider those at which P is true at the first world separately from those at which P is false at that world. Where P is true, the conditional is equivalent to its consequent $S \rightarrow W$. Since S is false, the probability of $S \rightarrow W$

[25]
$$\begin{aligned}\mathrm{P}(PSW) &= \mathrm{P}(PSWI) + \mathrm{P}(PSW\overline{I})\\ &= \mathrm{P}(I)\mathrm{P}(P|I)\mathrm{P}(S|IP)\mathrm{P}(W|SIP) + \mathrm{P}(\overline{I})\mathrm{P}(P|\overline{I})\mathrm{P}(S|\overline{I}P)\mathrm{P}(W|S\overline{I}P)\\ &= \mathrm{P}(I)\mathrm{P}(P|I)\mathrm{P}(S)\mathrm{P}(W|SI) + \mathrm{P}(\overline{I})\mathrm{P}(P|\overline{I})\mathrm{P}(S)\mathrm{P}(W|S\overline{I})\\ &\quad \text{[due to independence assumptions]}\\ &= 0.25 \times 0.45 \times 0.1 \times 0.9 + 0.75 \times 0.05 \times 0.1 \times 0.1 = 0.0105\end{aligned}$$

depends on $V^*(I)$: It is $\mathrm{P}(W|SI)$ if Harry is intelligent, and $\mathrm{P}(W|S\overline{I})$ if he is not. Thus:[26]

$$\mathbb{P}^*((P \to (S \to W)) \wedge P\overline{S}I) = \mathrm{P}(W|SI)\mathrm{P}(P\overline{S}I) \tag{62}$$

$$\mathbb{P}^*((P \to (S \to W)) \wedge P\overline{S}\,\overline{I}) = \mathrm{P}(W|S\overline{I})\mathrm{P}(P\overline{S}\,\overline{I}) \tag{63}$$

Finally, if both S and P are false at ω^*, the conditional is evaluated at the first P-subsequence at which the values of all non-descendants of $\mathrm{V}^*(P)$ are preserved. For instance, the value at a $\overline{P}\,\overline{S}IT$-sequence is that of the consequent at the first $P\overline{S}IT$-subsequence. Notice that in this case $V^*(W)$ does not affect the value of the overall sentence: Since S is false, the value of the sentence is that of $V^*(W)$ at the *next* subsequence at which S is true and $V^*(S)$'s causal non-descendants are preserved; $V^*(W)$ is not among those non-descendants. The two cases for I and $\overline{I}$ are given in (64) and (65).[27]

$$\mathbb{P}^*((P \to (S \to W)) \wedge \overline{P}\,\overline{S}I) = \mathrm{P}(W|SI)\mathrm{P}(\overline{P}\,\overline{S}I) \tag{64}$$

$$\mathbb{P}^*((P \to (S \to W)) \wedge \overline{P}\,\overline{S}\,\overline{I}) = \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{P}\,\overline{S}\,\overline{I}) \tag{65}$$

This exhausts all cases, and the overall probability of the sentence is given in (66).[28]

$$\begin{aligned} &\mathbb{P}^*((P \to (S \to W))) \\ &\quad = \mathrm{P}(SW) + \mathrm{P}(W|SI)\mathrm{P}(\overline{S}I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{S}\,\overline{I}) \end{aligned} \tag{66}$$

Notice that the right-hand side of (66) is already familiar from Section 4.1 above: It is just the probability (under the local interpretation) of the con-

[26] $$\begin{aligned} &\mathbb{P}^*((P \to (S \to W)) \wedge P\overline{S}IT) \\ &\quad = \mathrm{P}(P\overline{S}IT)\mathrm{P}(PSITW) + \mathrm{P}(P\overline{S}IT)\mathrm{P}(\overline{PSIT})\mathrm{P}(PSITW) \\ &\qquad + \mathrm{P}(P\overline{S}IT)\mathrm{P}(\overline{PSIT})^2\mathrm{P}(PSITW) + \ldots \\ &\quad = \mathrm{P}(P\overline{S}IT)\mathrm{P}(PSITW)/\mathrm{P}(PSIT) = \mathrm{P}(W|PSIT)\mathrm{P}(P\overline{S}IT) \\ &\quad = \mathrm{P}(W|SI)\mathrm{P}(P\overline{S}IT) \end{aligned}$$

For the last step, recall that $\mathrm{P}(W|PSI) = \mathrm{P}(W|SI)$ by stochastic independence. A similar argument yields $\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}I\overline{T})$ for the case that T is false, thus we have $\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}I)$ in (62). The case of $P\overline{S}\,\overline{I}$ in (63) is similar.

[27] $$\begin{aligned} &\mathbb{P}^*((P \to (S \to W)) \wedge \overline{P}\,\overline{S}IT) \\ &\quad = \mathrm{P}(\overline{P}\,\overline{S}IT)\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}IT) + \mathrm{P}(\overline{P}\,\overline{S}IT)\mathrm{P}(\overline{P\overline{S}IT})\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}IT) \\ &\qquad + \mathrm{P}(\overline{P}\,\overline{S}IT)\mathrm{P}(\overline{P\overline{S}IT})^2\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}IT) + \ldots \\ &\quad = \mathrm{P}(\overline{P}\,\overline{S}IT)\mathrm{P}(W|SI)\mathrm{P}(P\overline{S}IT)/\mathrm{P}(P\overline{S}IT) = \mathrm{P}(W|SI)\mathrm{P}(\overline{P}\,\overline{S}IT) \end{aligned}$$

A similar argument yields $\mathrm{P}(W|SI)\mathrm{P}(\overline{P}\,\overline{S}I\overline{T})$ for the case that T is false, thus we have $\mathrm{P}(W|SI)\mathrm{P}(\overline{P}\,\overline{S}I)$ in (64). The case of (65) is similar.

[28] $$\begin{aligned} &\mathbb{P}^*((P \to (S \to W))) \\ &\quad = \mathrm{P}(SW) + \mathrm{P}(W|SI)[\mathrm{P}(P\overline{S}I) + \mathrm{P}(\overline{P}\,\overline{S}I)] + \mathrm{P}(W|S\overline{I})[\mathrm{P}(P\overline{S}\,\overline{I}) + \mathrm{P}(\overline{P}\,\overline{S}\,\overline{I})] \end{aligned}$$

sequent $S \to W$. Indeed, the causal independence between P and S implies that (56) is equivalent to its consequent:

(67) $$\mathbb{V}(P \to (S \to W))(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SW)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{W})(\omega) = 1 \\ \mathrm{P}(W|SI) \text{ if } \mathrm{V}(\overline{S}I)(\omega) = 1 \\ \mathrm{P}(W|S\overline{I}) \text{ if } \mathrm{V}(\overline{S}\,\overline{I})(\omega) = 1 \end{cases}$$

Is this a sensible result? About the probability of 0.34 in (58), I said that it was too low. If that is right, then the assignment in (67) should be even worse, for recall that $\mathrm{P}(W|S) = 0.3$. Have we gained anything?

To see that we have, recall that so far I have only discussed the *local* interpretation of the sentence. We will see in a moment that its global interpretation is very much in line with the intuitions I alluded to in connection with (58) above.

First, however, notice that the conditional does have a reading under which its probability is low. The distribution of values in (67) captures the intuition that the conditional *'If Harry is selected, he will win'* is causally independent of his performance in the test.[29] This is a feature of this particular scenario with its attendant causal independence assumptions. The probability of 0.3 is the expectation of the sentence under these assumptions—in particular, none of the variables in question are causally affected by Harry's passing the test. To make this explicit, the sentence may be given a concessive paraphrase:

(68) Even if Harry passes the test, he will still not win if he is selected (because I don't think he is intelligent, and his passing the test does not make him so).

The conditional certainly has this reading, although it may not be the most prominent one unless it is brought out as in the paraphrase. The more prominent reading corresponds to the *global* probability $\mathrm{P}^*(S \to W|P)$, where $S \to W$ is again interpreted locally. This can be calculated using the results above and standard Bayesian reasoning:[30]

(69) $$\begin{aligned}&\mathbb{P}^*((S \to W) \wedge P)\\ &= \mathrm{P}(PSW) + \mathrm{P}(W|SI)\mathrm{P}(P\overline{S}I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(P\overline{S}\,\overline{I})\end{aligned}$$

[29]In the terminology of Bayesian networks, the set $\{V(S), V(W)\}$ is *d-separated* from $V(P)$ since it is unknown whether Harry is intelligent. I leave a detailed exploration of the relationship between the notion of d-separation and the interpretation of conditionals proposed here to another occasion.

[30] $P(I|P) = P(P|I)P(I)/P(P) = 0.9 \times 0.25/0.3 = 0.75$. Since $\overline{S}$ is independent of both I and P, $P(\overline{S}I|P) = P(\overline{S}|IP)P(I|P) = P(\overline{S})P(I|P) = 0.9 \times\ 0.75 = 0.675$. Similarly, $P(\overline{S}\,\overline{I}|P) = 0.9 \times 0.25 = 0.225$.

(70)
$$\begin{aligned}\mathbb{P}^*(S \to W|P) &= \mathrm{P}((S \to W) \wedge P)/\mathrm{P}(P)\\ &= \mathrm{P}(SW|P) + \mathrm{P}(W|SI)\mathrm{P}(\overline{S}I|P) + \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{S}\,\overline{I}|P)\\ &= 0.07 + 0.9 \times 0.675 + 0.1 \times 0.225 = 0.7\end{aligned}$$

This, I believe, is the intuitively correct probability for the conditional on its most prominent reading. It accounts for the fact that the hypothesized passing of the test is evidence for Harry's being intelligent: In (70), $\mathrm{P}(\overline{S}I|P)$ is greater than $\mathrm{P}(\overline{S}\,\overline{I}|P)$, while in the original distribution, $\mathrm{P}(\overline{S}I)$ was less than $\mathrm{P}(\overline{S}\,\overline{I})$.

Notice, incidentally, that (70) resembles the original formula in (57) rather closely. The only difference is that here, two classes of $\overline{S}$-worlds are distinguished, depending on whether Harry is intelligent or not. Notice also that as in the case of simple conditionals, in making the distinction between local and global interpretations, we are not committed to different assignments of values to the conditional: What changes are the respective probabilities that it takes those values.

5.5.1 The Match

Consider now another example of right-nested conditionals, which was first discussed by Edgington [8]:

(71) a. If the match is wet, then if you strike it, it will light.
b. $W \to (S \to L)$

Kaufmann [22] discusses this example at some length in justifying the causally sensitive assignment of values to conditionals at non-antecedent worlds. The formal apparatus Kaufmann employs there is simpler than the one proposed here and does not extend to conditionals with conditional antecedents. In addition, Kaufmann limits the discussion to a particular interpretation of the probability measure as "objective chance," thus avoiding the added complexity of the abductive inference that leads to global probabilities. For these reasons, the example is worth reviewing in light of the present proposal. The scenario is the following.

(72) The probability that ...

a. it gets wet is "low" (0.1)
b. you strike it is 0.5
c. it lights given that you strike it and it is dry is "high" (0.9)
d. it lights given that you strike it and it is wet is "low" (0.1)

and the striking and the wetness are stochastically independent.

Intuitively, the probability of (71) should be low under these circumstances. However, this is not what the original formula for right-nested conditionals predicts. To see what its prediction is, we need to know that $P(SL|W) = 0.05$

and the conditional probability $P(L|S)$ that the match will light, given that it is struck, is 0.82.[31]

Notice also that $\mathrm{P}(\overline{S}|W) = \mathrm{P}(\overline{S})$ since S and W are independent. Then according to the original formula, the probability of (71) is as follows:

$$\text{(73)} \qquad \begin{aligned} \mathrm{P}^*(W \to (S \to L)) &= \mathrm{P}(SL|W) + \mathrm{P}(L|S)P(\overline{S}|W) \\ &= 0.05 + 0.82 \times 0.5 = 0.46 \end{aligned}$$

But intuitively, the probability should be lower than this. The source of the problem, Kaufmann [22] argues, lies in the distribution of the values of the consequent $S \to L$ under the original interpretation:

$$\text{(74)} \qquad \mathrm{V}(S \to L)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SL)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{L})(\omega) = 1 \\ 0.82 = \mathrm{P}(L|S) \text{ if } \mathrm{V}(\overline{S})(\omega) = 0 \end{cases}$$

The third line in particular, assigning $\mathrm{P}(L|S)$ uniformly at all $\overline{S}$-worlds, is problematic. Since I have already discussed this example above in Section 4.1, I will not reiterate the argument here. With the revised value assignment, the problem does not arise, provided that the causally relevant variables in this case stand in the relation discussed earlier. Thus whether the match lights depends both on whether it is wet and whether it is struck, but no causal connection holds between those latter two variables. Under these assumptions, the value assignment at worlds in Ω becomes (75).

$$\text{(75)} \qquad \mathbb{V}(S \to L)(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(SL)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(S\overline{L})(\omega) = 1 \\ 0.1 = \mathrm{P}(L|WS) \text{ if } \mathrm{V}(\overline{S}W)(\omega) = 1 \\ 0.9 = \mathrm{P}(L|\overline{W}S) \text{ if } \mathrm{V}(\overline{S}\,\overline{W})(\omega) = 1 \end{cases}$$

Turning to the sentence $W \to (S \to L)$, we can see now that it is a somewhat peculiar special case, in that $V(W)$, the one relevant background factor which, jointly with $V(S)$, determines the expectation of $\mathrm{V}(L)$, is also the antecedent of the conditional. This simplifies matters, but it also obscures the consequences of the causally sensitive value assignment, since as a result, the local and global probabilities are the same. To see this, recall that the values for this sentence are defined as follows.

$$\text{(76)} \qquad \begin{aligned} \mathbb{V}^*(W \to (S \to L))(\omega^*) &= \mathbb{V}^*(S \to L)(\omega^*(n)) \\ &\text{for the least } n \text{ such that } \mathrm{V}^*(W)(\omega^*(n)) = 1 \\ &\text{and } \mathrm{V}^*(S)(w^*(n)) = \mathrm{V}^*(S)(w^*) \end{aligned}$$

[31] Since the wetness and the striking are independent, $\mathrm{P}(W|S) = \mathrm{P}(W)$ and $\mathrm{P}(\overline{W}|S) = \mathrm{P}(\overline{W})$.
Thus $\mathrm{P}(SL|W) = \mathrm{P}(L|SW)\mathrm{P}(S|W) = \mathrm{P}(L|SW)\mathrm{P}(S) = 0.1 \times 0.5 = 0.05$
and
$$\begin{aligned} \mathrm{P}(L|S) &= \mathrm{P}(LS)/\mathrm{P}(S) = [\mathrm{P}(LSW) + \mathrm{P}(LS\overline{W})]/\mathrm{P}(S) \\ &= [\mathrm{P}(L|SW)\mathrm{P}(W|S)\mathrm{P}(S) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W}|S)\mathrm{P}(S)]/\mathrm{P}(S) \\ &= \mathrm{P}(L|SW)\mathrm{P}(W) + \mathrm{P}(L|S\overline{W})\mathrm{P}(\overline{W}) = 0.1 \times 0.1 + 0.9 \times 0.9 = 0.82 \end{aligned}$$

Clearly the conditional is equivalent to L at sequences in which WS is true at the first world. Hence (77). At sequences at which the match is dry and struck, the conditional is evaluated at the first world at which it is wet and struck, giving (78).[32]

(77) $$\mathbb{P}^*((W \to (S \to L)) \wedge WS) = \mathrm{P}(WSL)$$

(78) $$\mathbb{P}^*((W \to (S \to L)) \wedge \overline{W}S) = \mathrm{P}(L|WS)\mathrm{P}(\overline{W}S)$$

Next, at $W\overline{S}$-sequences, the value of the conditional is that of the consequent $S \to L$ at the first WS-subsequence[33] and at $\overline{W}\,\overline{S}$-sequences, it is that of the consequent at the first $W\overline{S}$-subsequence.[34] The expectations of those values are (79) and (80), respectively.

(79) $$P^*((W \to (S \to L)) \wedge W\overline{S}) = P(L|WS)P(W\overline{S})$$

(80) $$P^*((W \to (S \to L)) \wedge \overline{W}\,\overline{S}) = P(L|WS)P(\overline{W}\,\overline{S})$$

Thus finally, the overall probability of the sentence is the following.[35]

(81) $$\mathbb{P}^*(W \to (S \to L)) = \mathrm{P}(L|WS) = 0.1$$

The distribution of "unstarred" values over worlds in Ω that corresponds to this new assignment is (82).

(82) $$\mathrm{V}(W \to (S \to L))(\omega) = \begin{cases} 1 \text{ if } \mathrm{V}(WSL)(\omega) = 1 \\ 0 \text{ if } \mathrm{V}(WS\overline{L})(\omega) = 1 \\ 0.1 = \mathrm{P}(L|WS) \\ \qquad \text{if } \mathrm{V}(W)(\omega) = 0 \text{ or } \mathrm{V}(S)(\omega) = 0 \end{cases}$$

[32] $\mathbb{P}^*((W \to (S \to L)) \wedge \overline{W}S)$
$= \mathrm{P}(\overline{W}S)\mathrm{P}(WSL) + \mathrm{P}(\overline{W}S)\mathrm{P}(\overline{WS})\mathrm{P}(WSL) + \mathrm{P}(\overline{W}S)\mathrm{P}(\overline{WS})^2\mathrm{P}(WSL) + \ldots$
$= \mathrm{P}(\overline{W}S)\mathrm{P}(WSL)/\mathrm{P}(WS) = \mathrm{P}(L|WS)\mathrm{P}(\overline{W}S)$

[33] $\mathbb{P}^*((W \to (S \to L)) \wedge W\overline{S})$
$= \mathrm{P}(W\overline{S})\mathrm{P}(WSL) + \mathrm{P}(W\overline{S})\mathrm{P}(\overline{WS})\mathrm{P}(WSL) + \mathrm{P}(W\overline{S})\mathrm{P}(\overline{WS})^2\mathrm{P}(WSL) + \ldots$
$= \mathrm{P}(W\overline{S})\mathrm{P}(WSL)/\mathrm{P}(WS) = \mathrm{P}(L|WS)\mathrm{P}(W\overline{S})$

[34] $\mathbb{P}^*((W \to (S \to L)) \wedge \overline{W}\,\overline{S})$
$= \mathrm{P}(\overline{W}\,\overline{S}) \times (79) + \mathrm{P}(\overline{W}\,\overline{S})\mathrm{P}(\overline{W\overline{S}}) \times (79) + \mathrm{P}(\overline{W}\,\overline{S})\mathrm{P}(\overline{W\overline{S}})^2 \times (79) + \ldots$
$= \mathrm{P}(\overline{W}\,\overline{S})\mathrm{P}(L|WS)\mathrm{P}(W\overline{S})/\mathrm{P}(W\overline{S})$
$= \mathrm{P}(L|WS)\mathrm{P}(\overline{W}\,\overline{S})$

[35] $\mathbb{P}^*(W \to (S \to L)) = \mathrm{P}(WSL) + \mathrm{P}(L|WS)[\mathrm{P}(\overline{W}S) + \mathrm{P}(W\overline{S}) + \mathrm{P}(\overline{W}\,\overline{S})]$
$= \mathrm{P}(L|SW)[\mathrm{P}(WS) + \mathrm{P}(\overline{W}S) + \mathrm{P}(W\overline{S}) + \mathrm{P}(\overline{W}\,\overline{S})]$

The expectation of these values, given in (81), corresponds to the local reading of the conditional. The global reading is again obtained by conditioning the various events in which the conditional has non-zero values on the antecedent W. This results in (83):

(83)
$$\begin{aligned}\mathbb{P}^*(W \to (S \to L)) &= \mathrm{P}(SL|W) + \mathrm{P}(L|WS)\mathrm{P}(\overline{S}|W)\\ &= 0.05 + 0.1 \times 0.5 = 0.1\end{aligned}$$

It is now clear in what sense the example is degenerate. The probabilities of the sentence on the local and global readings coincide: While the formula in (83) was derived as the local reading, the global reading comes down to the same.[36]

5.6 Left-nested Conditionals

The final problem I want to discuss concerns conditionals with conditional antecedents. The sentence in question is (25d), repeated here as (84).

(84) a. If Harry wins if he is selected for the show, he will pass the test.
b. $(S \to W) \to P$

Unlike the right-nested conditional in the previous section, this sentence does not involve a local/global distinction. It is useful to start by clarifying our intuitions as to what the value of the conditional should be. First, Harry can only pass the test if he takes it, and the probability that he takes it is 0.5 and independent of the other probabilities. Thus the probability of (84) will not exceed 0.5. It should, however, be significantly higher than 0.15, the unconditional probability that Harry will pass. After all, the antecedent is intuitively correlated with the consequent by providing evidence that Harry is intelligent.

With this in mind, we see that the probability under Jeffrey's original account is rather low: [37]

(85)
$$\begin{aligned}\mathrm{P}^*((S \to W) \to P) &= \mathrm{P}(P|SW)\mathrm{P}(S) + \mathrm{P}(\overline{S}P)\\ &= 0.35 \times 0.1 + 0.135 = 0.17\end{aligned}$$

[36] Simply note that the right-hand side in (83) is equivalent to $\mathrm{P}(SL|W) + \mathrm{P}(L|WS)\mathrm{P}(\overline{S}W|W) + \mathrm{P}(L|\overline{W}S)\mathrm{P}(\overline{S}\,\overline{W}|W)$, which corresponds to the formula in (70); here $V^*(W)$ is the causal background variable that corresponds to $V^*(I)$ in (70).

[37] Recall from Footnote 25 that $\mathrm{P}(PSW) = .0105$. Furthermore, $\mathrm{P}(SW) = \mathrm{P}(S)\mathrm{P}(W|S) = 0.1 \times 0.3$. Thus $\mathrm{P}(P|SW) = 0.0105/0.03 = 0.35$. Also, $\mathrm{P}(\overline{S}P) = \mathrm{P}(\overline{S})\mathrm{P}(P|\overline{S}) = \mathrm{P}(\overline{S})\mathrm{P}(P) = 0.9 \times 0.15 = 0.135$.

How does the revised value assignment fare? We already had occasion to calculate some of the required probabilities in Footnote 17 above:[38]

(86)
$$\begin{aligned}\mathbb{P}^*(P|S \to W) &= \frac{\mathbb{P}^*((S \to W) \wedge P)}{\mathbb{P}^*(S \to W)} \\ &= \frac{\mathrm{P}(PSW) + \mathrm{P}(W|SI)\mathrm{P}(P\overline{S}I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(P\overline{S}\,\overline{I})}{\mathrm{P}(SW) + \mathrm{P}(W|SI)\mathrm{P}(\overline{S}I) + \mathrm{P}(W|S\overline{I})\mathrm{P}(\overline{S}\,\overline{I})} \\ &= \frac{0.0105 + 0.9 \times 0.10125 + 0.1 \times 0.03375}{0.3} \\ &= 0.105/0.3 = 0.35\end{aligned}$$

Recall once again that there is only a 0.5 probability that Harry will take the test and even get a chance to pass it. In view of this fact, I believe, the result in (86) is intuitively plausible.

It is instructive to compare these values to those of the right-nested conditional above. The old and new probabilities in (85) and (86) are exactly half of the corresponding probabilities in (58) and (70). This is due to the fact that the probability that Harry takes the test is 0.5. The reader may care to verify that the expressions in (85) and (86) equal their above counterparts of 0.34 and 0.7 if the probability distribution is conditionalized on T.

Unlike in the case of right-nested conditionals, however, here we do not have a local probability corresponding to (66) above, since conditional antecedents are not assumed to participate in causal relations.

5.6.1 Alma

Jeffrey [20], in discussing left-nested conditionals, mentioned the example in (87a). His formula for calculating the probabilities of these sentences is that in Theorem 5, repeated here as (87b).

(87) a. If Alma succeeds if she tries, she's able.
b. $\mathrm{P}((T \to S) \to A) = \mathrm{P}(A|TS)\mathrm{P}(T) + \mathrm{P}(\overline{T}A)$

Jeffrey did not devote much discussion to this sentence; in fact, he did not even claim explicitly that the probability assigned to it by (87b) is intuitively right. Edgington [8] took up the example and argued that it is not:

> Suppose I accept the sentence. The only part of the formula to which I seem constrained to give a high value is $\mathrm{P}(A|TS)$. I needn't think it likely that she'll try; nor need I think it likely that she's able if she doesn't try — I might well think, on the contrary, that if she doesn't try, she's probably not able. But then, the whole formula can take a low value. (p. 202)

[38] Since P entails T, $\mathrm{P}(P\overline{S}I) = \mathrm{P}(TP\overline{S}I)$ and $\mathrm{P}(P\overline{S}\,\overline{I}) = \mathrm{P}(TP\overline{S}\,\overline{I})$.

To illustrate, Edgington supplied the following probabilities.

(88) S Alma succeeds
T Alma tries $\mathrm{P}(T) = 0.2$
A Alma is able $\mathrm{P}(A|TS) = 1$
$\mathrm{P}(A|\overline{T}) = 0.2$

Here $\mathrm{P}(A|TS)$ is as high as it can be. However, Edgington argues, the probability according to (87b) is too low:

(89)
$$\begin{aligned}\mathrm{P}^*((T \to S) \to A) =& \mathrm{P}(A|TS)P(T) + \mathrm{P}(A|\overline{T})\mathrm{P}(\overline{T})\\ =& 1 \times 0.2 + 0.2 \times 0.8 = 0.36\end{aligned}$$

Now, there are again some peculiarities, both in the example and in Edgington's argument, which merit some discussion. Firstly, the sentence, taken out of context, is semantically drastically underspecified. It is not clear what Alma is supposed to try and succeed at in the antecedent, nor what ability of hers is referred to in the consequent. This is significant, since lacking these specifications, one is easily biased towards an interpretation according to which the activity that Alma attempts in the antecedent is the same which she is (or is not) able to carry out in the consequent, as in (90):

(90) If Alma succeeds in cleaning the window if she tries to clean the window, she is an able window cleaner.

Under this interpretation, it is reasonable to assume that the antecedent depends causally on the consequent — that is, her ability determines whether she will succeed if she tries. Thus the consequent figures as an independent causal factor in the interpretation of the antecedent.

If we make this assumption about the causal structure of the setting, then the causally sensitive interpretation of the conditional will assign the high probability that Edgington would deem correct in this case. It is easy to see why this should be the case: The value of the conditional at world sequences is that of A at the first final subsequence at which $S \to T$ is true. Let $\omega^*(n)$ be such a subsequence ($1 \leq n$). The antecedent $S \to T$ may be true at $\omega^*(n)$ because ST is true at $\omega^*[n]$. In that case, A is also true at $\omega^*[n]$ with probability 1. Otherwise, $\omega^*[n]$ is a $\overline{S}$-world and T is true at the next relevant final subsequence in $\omega^*(n)$ at which S is true. Call this sequence $\omega^*(m)$. Since A is a causally independent background variable, it has the same value at $\omega^*(m)$ that it has at $\omega^*(n)$. But since $\omega^*[m]$ is an ST-world, A is true there with probability 1.

Indeed, in Edgington's scenario, with the added assumption about the causal role of A, the conditional $(S \to T) \to A$ has probability 1 under the revised value assignment. This can be shown using the formula we derived in (86) above.[39]

The causally sensitive assignment delivers the desired probability, but it does so only under the assumption that a certain causal structure obtains in the scenario. Lacking that structure, the same probabilities may give rise to a lower probability for the sentence. This possibility is still at odds with Edgington's contention that a high $\mathrm{P}(A|TS)$ is not only necessary but also *sufficient* for a high probability of the conditional. (Recall the statement that in giving a high probability to $(T \rightarrow S) \rightarrow A$, $\mathrm{P}(A|TS)$ is "the only part of (87b) to which I am constrained to give a high value.") Before concluding this section, I will explain why I do not share this judgment, although it may seem plausible in this particular scenario. A closer look at this matter sheds some more light on the way causal relationships can confound judgments about probabilities.

Consider the sentence (87) in the context of the following scenario. A job opening is announced for which Alma would be the perfect candidate. It is virtually certain that if she tries (to get the job), she will succeed (in getting it): $\mathrm{P}(T \rightarrow S) = 0.99$. Moreover, the job pays handsomely, so if she tries and succeeds in getting it, she will be able to order the car of her dreams: $\mathrm{P}(A|TS) = 1$. Is this sufficient for (87) to have a high probability? I believe it is not, because in this scenario, our strong belief in the antecedent $T \rightarrow S$ does not imply that we assign a high probability to the consequent A. To see this, add one further piece of information: The job would require her to relocate to a remote area, far away from her friends and relatives, and therefore, knowing her, we think it rather unlikely that she will apply. This last assumption is consistent with all the earlier ones, in particular with our strong belief in both $\mathrm{P}(T \rightarrow S)$ and $\mathrm{P}(A|TS)$. But now we certainly would not conclude that Alma will be able to order the car of her dreams!

There are, to be sure, subtle differences between Jeffrey's original sentence and the one I just used. The main difference concerns the form of the predicate in the consequent ('*she is able*' vs. '*she will be able*'). The temporal relations between the events in the three clauses may well be a crucial element in explaining the fact that a high $\mathrm{P}(A|TS)$ seems sufficient for a high probability of (87a), but not its variant in the job application scenario. But this only shows that Edginton's argument derives its intuitive appeal from special properties of one particular example. It does not hold as a generalization about the logic of left-nested conditionals. The question remains, of course, what is responsible for its validity in those cases in which it does hold. But this question leads

[39] First, notice that $\mathrm{P}(S|T\overline{A}) = \mathrm{P}(ST\overline{A})/\mathrm{P}(T\overline{A}) = \mathrm{P}(\overline{A}|TS)\mathrm{P}(TS)/\mathrm{P}(T\overline{A}) = 0 \times \mathrm{P}(TS)/\mathrm{P}(T\overline{A}) = 0$. Recall also that $\mathrm{P}(T \rightarrow S) = \mathrm{P}(S|TA)\mathrm{P}(A) + \mathrm{P}(S|T\overline{A})\mathrm{P}(\overline{A})$. Thus:

$$\mathbb{P}^*(A|T \rightarrow S) = \frac{\mathrm{P}(ATS) + \mathrm{P}(S|TA)\mathrm{P}(A\overline{T}A) + \mathrm{P}(S|T\overline{A})\mathrm{P}(A\overline{T}\,\overline{A})}{\mathrm{P}(S|TA)\mathrm{P}(A) + \mathrm{P}(S|T\overline{A})\mathrm{P}(\overline{A})}$$
$$= \frac{\mathrm{P}(ATS) + \mathrm{P}(S|TA)\mathrm{P}(\overline{T}A)}{\mathrm{P}(S|TA)\mathrm{P}(A)} = \frac{\mathrm{P}(S|TA)[\mathrm{P}(TA) + \mathrm{P}(\overline{T}A)]}{\mathrm{P}(S|TA)P(A)} = 1$$

beyond the scope of this paper. What I hope to have shown is that left-nested conditionals do sometimes have a low probability in a scenario like (88). Together with the explanation for the fact that (87a) does not have a low probability, this is a significant step forward.

6 Conclusion

This paper has proposed a refinement of the probabilistic theory of compounds of conditionals, and to the extent that it is successful, it has thereby also offered evidence for that theory. The criticism that was leveled against earlier accounts was a crucial motivating force behind the development of the present proposal. Some of its elements, in particular the reliance on causal relations, will no doubt invite criticism of their own. On the other hand, the proposal fits organically into a larger theoretical picture of the relationship between counterfactual and indicative conditionals, as well as the variable ways in which the latter are evaluated [21–23]. Causality and the distinction between "local" and "global" readings are already integral parts of that theory.

Acknowledgements I am grateful to an anonymous reviewer for detailed critical comments on an earlier version of this paper. All remaining errors and misrepresentations are my own. This work was supported in part by the Japan Society for the Promotion of Science (JSPS, "The Logic of Everyday Inference and its Linguistic Forms: With Special Reference to Quantificational Expressions, Conditionals, and Modal Expressions").

References

1. Adams, E. (1965). The logic of conditionals. *Inquiry, 8*, 166–197.
2. Adams, E. (1970). Subjunctive and indicative conditionals. *Foundations of Language, 6*, 89–94.
3. Adams, E. (1975). *The logic of conditionals*. Dordrecht: Reidel.
4. Adams, E. (1998). *A primer of probability logic*. Stanford: CSLI.
5. Appiah, A. (1984). Jackson on the material conditional. *Australasian Journal of Philosophy, 62*(1), 77–81.
6. Appiah, A. (1985). *Assertion and conditionals*. Cambridge: Cambridge University Press.
7. Cowell, R. G., Dawid, A. P., Lauritzen, S. L., & Spiegelhalter,D. J. (1999). *Probabilistic networks and expert systems*. Berlin: Springer.
8. Edgington, D. (1991). The mystery of the missing matter of fact. In *Proceedings of the Aristotelian society* (pp. 185–209). London: The Aristotelian Society.
9. Edgington, D. (1995). On conditionals. *Mind, 104*(414), 235–329.
10. Ellis, B. (1984). Two theories of indicative conditionals. *Australasian Journal of Philosophy, 62*(1), 50–66.
11. Gibbard, A. (1981a). Indicative conditionals and conditional probability: Reply to pollock. In W. L. Harper, R. Stalnaker, & G. Pearce (Eds.), *Ifs: Conditionals, belief, decision, chance, and time* (pp. 253–255). Dordrecht: Reidel.
12. Gibbard, A. (1981b). Two recent theories of conditionals. In W. L. Harper, R. Stalnaker, & G. Pearce (Eds.), *Ifs: Conditionals, belief, decision, chance, and time* (pp. 211–247). Dordrecht: Reidel.
13. Glymour, C. (2001). *The mind's arrows: Bayes nets and graphical causal models in psychology*. Cambridge: MIT.

14. Hájek, A., & Hall, N. (1994). The hypothesis of the conditional construal of conditional probability. In E. Eells, & B. Skyrms (Eds.), *Probabilities and conditionals: belief revision and rational decision* (pp. 75–110). Cambridge: Cambridge University Press.
15. Halpern, J. Y. (2003). *Reasoning about uncertainty*. Cambridge: MIT.
16. Jackson, F. (1979). On assertion and indicative conditionals. *Philosophical Review, 88*, 565–589.
17. Jackson, F. (1984). Two theories of indicative conditionals: Reply to Brian Ellis. *Australasian Journal of Philosophy, 62*(1), 67–76.
18. Jackson, F. (1987). *Conditionals*. Oxford: Basil Blackwell.
19. Jeffrey, R. C. (1964). If. *Journal of Philosophy, 61*, 702–703.
20. Jeffrey, R. C. (1991). Matter-of-fact conditionals. In *The Symposia Read at the Joint Session of the Aristotelian Society and the Mind Association at the University of Durham* (Suppl. Vol. 65, pp. 161–183). London: The Aristotelian Society.
21. Kaufmann, S. (2004). Conditioning against the grain: Abduction and indicative conditionals. *Journal of Philosophical Logic, 33*(6), 583–606.
22. Kaufmann, S. (2005). Conditional predictions: A probabilistic account. *Linguistics and Philosophy, 28*(2), 181–231.
23. Kaufmann, S., Winston, E., & Zutty, D. (2004). Local and global interpretations of conditionals. *Presented at the Eighth Symposium on Logic and Language (LoLa8)*. Debrecen, Hungary.
24. Kyburg, H. E., man Teng, C., Wheeler, G. (2005). Conditionals and consequences. In *Proceedings of the Fourth International Workshop on Computational Models of Scientific Reasoning and Applications (CMSRA IV)*. Lisbon: Universidade Nova de Lisboa.
25. Lance, M. (1991). Probabilistic dependence among conditionals. *Philosophical Review, 100*, 269–276.
26. Lewis, D. (1976). Probabilities of conditionals and conditional probabilities. *Philosophical Review, 85*, 297–315.
27. Lewis, D. (1979). Counterfactual dependence and time's arrow. *Noûs, 13*, 455–476.
28. Lewis, D. (1986). Postscript to "Probabilities of conditionals and conditional probabilities." In *Philosophical Papers* (Vol. 2, pp. 152–156). Oxford University Press.
29. McDermott, M. (1996). On the truth conditions of certain 'if'-sentences. *Philosophical Review, 105*, 1–37.
30. McGee, V. (1989). Conditional probabilities and compounds of conditionals. *The Philosophical Review, 98*(4), 485–541.
31. McGee, V. (2000). To tell the truth about conditionals. *Analysis, 60*, 107–111.
32. Pearl, J. (2000). *Causality: Models, reasoning, and inference*. Cambridge: Cambridge University Press.
33. Pollock, J. L. (1981). Indicative conditionals and conditional probability. In W. L. Harper, R. Stalnaker, & G. Pearce (Eds.), *Ifs: Conditionals, belief, decision, chance, and time* (pp. 249–252). Dordrecht: Reidel.
34. Ramsey, F. P. (1929). General propositions and causality. In D. H. Mellor (Ed.), *Philosophical papers: F. P. Ramsey*. Cambridge: Cambridge University Press (pp. 145–163).
35. Russell, B. (1903). *Principles of mathematics*. Cambridge: Cambridge University Press.
36. Sanford, D. (1989). *If P, then Q: Conditionals and the foundations of reasoning*. London: Routledge.
37. Spirtes, P., Glymour, C., & Scheines, R. (2000). *Causation, prediction, and search*. Cambridge: MIT.
38. Spohn, W. (2001). Bayesian nets are all there is to causal dependence. In M. C. Galavotti, P. Suppes, & D. Costantini (Eds.), *Stochastic causality* (pp. 157–172). Stanford: CSLI.
39. Stalnaker, R. (1970). Probability and conditionals. *Philosophy of Science, 37*, 64–80.
40. Stalnaker, R., & Jeffrey, R. (1994). Conditionals as random variables. In E. Eells, & B. Skyrms (Eds.), *Probabilities and conditionals: Belief revision and rational decision* (pp. 31–46). Cambridge: Cambridge University Press.

41. van Fraassen, B. C. (1976). Probabilities of conditionals. In W. L. Harper, R. Stalnaker, & G. Pearce (Eds.), *Foundations of probability theory, statistical inference, and statistical theories of science*. *The University of Western Ontario series in philosophy of science* (Vol. 1, pp. 261–308). Dordrecht: D. Reidel.
42. Woods, M. (1997). *Conditionals*. Clarendon: Clarendon.
43. Woodward, J. (2001). Probabilistic causality, direct causes, and counterfactual dependence. In M. C. Galavotti, P. Suppes, & D. Costantini (Eds.), *Stochastic causality* (pp. 39–63). Stanford: CSLI.