Dynamic Semantics: Day 2

Stefan Kaufmann

Kyoto University
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1 Introduction

1.1 What we will be talking about

Modality

(1) a. It might be raining.
b. It's not raining.
c. #It might be raining.

(2) a. It's not raining.
b. #It might be raining.

Modal subordination

(3) a. If John bought a book, he'll be home reading it by now.
b. It'll be a murder mystery.

(4) a. A wolf might come in.
b. It would eat you first.

(5) *(Mary and Beth approaching Mary's home at night; the house is dark)*

M: "My husband isn't home yet."
B: "How do you know?"
M: "The light would be turned on."

Conditional questions

(6) a. If Alfonso comes to the party, will Joanna come as well?
b. Yes, she will.
c. Alfonso isn't coming.

(7) a. A wolf might come in.
b. Do you think it would eat me?

2 Groenendijk et al. (1996)

The goal of this paper is to extend previous dynamic accounts to include world knowledge and modal expressions in the coverage. The idea was not entirely new at the time (recall ouf discussion of Heim, 1983b), but this paper explores the matter more in-depth.

2.1 Two kinds of information

In any discourse, two kinds of information are simultaneously being conveyed:

1. Information about the world
   What the world is like – facts, propositions – extralinguistic content.
   As usual, this is modeled with **possible worlds**

2. Information about the discourse
   Which individuals are being talked about and what is known or asserted said about them – linguistic content.
   The is modeled with **referent systems** (much like assignment functions, but slightly more complex).

The goal of the paper is to account for the **interaction** between the two kinds of information:

- Learning something about a discourse referent may eliminate possible worlds.
- Eliminating possible worlds may affect your beliefs about discourse referents.

Let's look at their system in some detail.

2.2 The formal setup

Referent systems

Readers of this paper sometimes complain about **referent systems**, finding the useless and complicated. But they are easy to get used to once you get your head around them.

- The basic idea behind referent systems is that GSV want to make it possible to "recycle" variables, just as it is possible to "recycle" pronouns in English.

  (8) a. A man$_i$ came in. He$_i$ was tall.
      b. John greeted the man$_i$. He$_j$ offered him$_i$ his$_j$ seat.

  In (9b), '*he*' refers to John, no longer to the man.

  ➨ English has only a small number of pronouns. GSV want a logic which relies on a small number of variables.

  Note: As before, this theory doesn't actually make predictions about the choice among possible coindexings for a given sentence. It only tells us which coindexings are and aren't possible.

- How they do it: Introduce an additional layer of representation between variables and the individuals they denote. These intermediate representations are natural numbers.

- Whenever a variable is introduced (by a quantifier), regardless of whether it has been introduced before or not, assign it to the least natural number not yet used.

$$\begin{array}{ccccc} \; & \; & \; & \; & \; \\ \hline 0 & 1 & 2 & 3 & 4\ldots \end{array} \; [x/0] \; \begin{array}{ccccc} x & & & & \\ \hline 0 & 1 & 2 & 3 & 4\ldots \end{array} \; [y/1] \; \begin{array}{ccccc} x & y & & & \\ \hline 0 & 1 & 2 & 3 & 4\ldots \end{array} \; [x/2] \; \begin{array}{ccccc} & y & x & & \\ \hline 0 & 1 & 2 & 3 & 4\ldots \end{array} \; [x/3] \; \begin{array}{ccccc} & y & & x & \\ \hline 0 & 1 & 2 & 3 & 4\ldots \end{array}$$

Formally:

i. A referent system is a function $r : V \mapsto \mathbb{N}$, where V is a finite set of variables.

Let a referent system with "range n" be one whose domain consists of the n numbers from 0 through $n - 1$. For instance, the r in (7) has range 1, and the one in (8) has range 3.

Thus if a referent system has range n, the number n is itself not in its range. That's why the following definition works.

ii. $r[x/n] = r'$ iff

(a) $dom(r') = dom(r) \cup \{x\}$;
(b) $range(r') = n + 1$;
(c) $r'(x) = n$; and
(d) $r'(y) = r(y)$ for all $y \neq x$.

Intuitively, $r[x/n]$ is is the result of either introducing or recycling variable x.

iii. r' is an *extension* of r, $r \leq r'$, iff

(a) $dom(r) \subseteq dom(r')$ (all variables in the domain of r are also in the domain of r');
(b) $n \leq n'$ (the greatest number associated with a variable in r' is at least as great as the greatest number associated with a variable in r);
(c) all variables in the domain of r are either still mapped to the same numbers in r', or mapped to "fresh" numbers that were not yet in the range of r;
(d) "fresh" variables not in the domain of r are mapped to "fresh" pegs not in the range of r.

Possibilities

At the center of the model theory are possibilities. Two main intuitions motivate GSV's approach, both of which we have encountered separately before:

1. Possibilities, basically, are **world-assignment pairs**, as in Heim (1983b).
   Basically, the pegs are the very same thing as the natural numbers in Heims' system.

2. New discourse referents are introduced with **random (re-)assignment**, as in Kamp (1981); Heim (1983a); Groenendijk et al. (1996).
   When a variable is either recycled or newly introduced, all of its possible assignments are multiplied out.

Formally, a possibility is a triple $\langle r, g, w \rangle$, where

i. r is a referent system (i.e., a function from some set of variables to some finite initial subset of the natural numbers).

ii. g is function from the range of r to individuals.

Let's stop here and look at the interaction between r and g. Suppose there are two individuals in the domain D of the model, a and b. In (9), we start with empty r and g (i.e., no discourse referents have been used yet).

(9)

0 1 2 3 ...

$[x/0]$

x
0 1 2 3 ...
a
b

$[y/1]$

x y
0 1 2 3 ...
a a
a b
b a
b b

$[x/3]$

y x	
0 1 2 3 ...	r
a a a	g_1
a a b	g_2
a b a	g_3
a b b	g_4
b a a	g_5
b a b	g_6
b b a	g_7
b b b	g_8

The rightmost table in (9) depicts eight possible assignments, each with the same referent system r but different assignments g from pegs to individuals. It should be obvious how the introduction or reassignment of a variable proceeds with random assignment to all possible values.

This example spells out all possible assignments for the pegs in use. Of course, not all of them assignments stay around, since facts about the discourse referents are also accumulated. Which assignments are eliminated in response to factual information is determined by **possible worlds**.

i. Let W be a set of possible worlds. The set I of **possibilities** based on domain D and universe W is the set of triples $\langle r, g, w \rangle$, where r is a referent system, g is a function from the range of r into D; and w is a world in W.

   Note: In the example in (9), we would have end up with a set of eight assignments like those in the rightmost table for each world. Thus there would $|W| \times 8$ possibilities after all and only the operations listed in (9) are carried out.

ii. For simplicity, for any possibility $i \in I$ and basic expression (i.e., variable or constant), we write $i(\alpha)$ to refer to the denotation of α in i.

   (a) If α is a variable in $dom(r)$, then $i(\alpha) = g(r(\alpha))$. (Otherwise undefined.)

   (b) If α is an individual constant, then $i(\alpha) = w(\alpha) \in D$.

   (c) If α is an n-ary predicate constant, then $i(\alpha) = w(\alpha) \subseteq D^n$. (That is, a set of n-tuples of individuals in D.)

Suppose for some world $w \in W$, $w(man) = \{b\}$. What happens if after the introduction of y in the model, the next thing we learn is that x is a man?

(10)

w
x y
0 1 2 3 ...
a a
a b
b a
b b

$man(x)$

w
x y
0 1 2 3 ...
b a
b b

$[x/3]$

w	
y x	
0 1 2 3 ...	r
b a a	g_5
b a b	g_6
b b a	g_7
b b b	g_8

➨ Once x has been reset with random assignment, it is longer restricted to men. However, the information that peg 0 refers to a man is preserved.

If there had been no men in w, then all possibilities with w would have been eliminated. To see this, let's look at some more definitions.

Information states

i. An **information state** is a set $s \subseteq I$ of possibilities that all have the same referent system. (The rules of interpretation ensure that the referent systems in these possibilities will always be the same.)

ii. The **(re-)assignment** of a variable to an individual $d \in D$ is modeled by a function $[x/d]$ on possibilities i and information states s:

(a) $i[x/d] = \langle r[x/range(r)], g[range(r)/d], w \rangle$

(b) $s[x/d] = \{i[x/d] | i \in s\}$

iii. A possibility $i' = \langle r', g', w' \rangle$ is an **extension** of a possibility $i = \langle r, g, w \rangle$ (written $i \leq i'$) iff:

(a) $r \leq r'$

(b) $g \subseteq g$

(c) $w = w'$

A state s' is an **extension** of a state s (written $s \leq s'$) iff every possibilities in s' is an extension of some possibility in s.

iv. Let s, s' be two information states such that s' is an extension of s, and let $i \in s, i' \in s'$. The:

(a) i' is a **descendant** of i iff $i \leq i'$.

(b) i **subsists** in i' iff i has at least one descendant in s'.

(c) s **subsists** in s' iff all $i \in s$ subsist in s'.

For instance, in the leftmost possibility depicted in (10), the assignments g_i with $g_i(0) = a$ do not subsist in the next state, but those with $g_i(0) = b$ do. The first state does not subsist in the second, but the second subsists in the third. (The last statement is only guaranteed to be true if $W = \{w\}$. We don't know anything about who is a man in other worlds.)

v. The state $\langle \emptyset, \emptyset, W \rangle$ is the state of (blissful) ignorance: No information about the world, no discourse information.

The state $\emptyset$ is the absurd state: No possibility left, no world compatible with the information. Everything is believed in $\emptyset$.

Update rules

Each formula $\varphi \in \mathcal{L}_{\mathcal{A}}$ denotes a function $[\![\varphi]\!]$ from states to states.
(*Note:* This is a relation between possibilities, but a function between states.)

$$
\begin{aligned}
s[\![P(t_1,\ldots,t_n)]\!] =& \{i \in s | \langle i(t_1),\ldots,i(t_n)\rangle \in i(P)\} \\
s[\![t_1 = t_2]\!] =& \{i \in s | i(t_1) = i(t_2)\} \\
s[\![\neg\varphi]\!] =& \{i \in s | i \text{ does not subsist in } s[\![\varphi]\!]\} \\
s[\![\varphi \wedge \psi]\!] =& s[\![\varphi]\!][\![\psi]\!] \\
s[\![\exists x\varphi]\!] =& \bigcup_{d\in D} (s[x/d][\![\varphi]\!]) \\
s[\![\Diamond\varphi]\!] =& \{i \in s | s[\![\varphi]\!] \neq \emptyset\}
\end{aligned}
$$

Comments:

- If φ contains no quantifiers, then $s[\![\varphi]\!] \subseteq s$.
- If φ contains a quantifier, then the number of possibilities in $s[\![\varphi]\!]$ may be greater than in s.
- The definition for $\exists x\varphi$ is a bit peculiar: Here we are not able to extract a definition for an operation $[\![\exists x]\!]$, as we could before.

  Let's see first what would have happened if we had executed $[\![\exists x\ man(x)]\!]$, rather than $[x/3]$, in the last step in (10).

$$
(11)\quad
\begin{array}{|l|}\hline w \\ \hline x\ \ y \\ \hline 0\ 1\ 2\ 3\ldots \\ \hline \text{b}\ \ \text{a} \\ \text{b}\ \ \text{b} \\ \hline\end{array}
[\![\exists x man(x)]\!] = \bigcup\left(
\begin{array}{|l|}\hline w \\ \hline \ \ \ \ y\ \ x \\ \hline 0\ 1\ 2\ 3\ldots \\ \hline \text{b}\ \ \text{a}\ \ \text{a} \\ \text{b}\ \ \text{b}\ \ \text{a} \\ \hline\end{array}
[\![man(x)]\!],
\begin{array}{|l|}\hline w \\ \hline \ \ \ \ y\ \ x \\ \hline 0\ 1\ 2\ 3\ldots \\ \hline \text{b}\ \ \text{a}\ \ \text{b} \\ \text{b}\ \ \text{b}\ \ \text{b} \\ \hline\end{array}
[\![man(x)]\!]\right)
$$

$$
= \bigcup\left(\emptyset,
\begin{array}{|l|}\hline w \\ \hline \ \ \ \ y\ \ x \\ \hline 0\ 1\ 2\ 3\ldots \\ \hline \text{b}\ \ \text{a}\ \ \text{b} \\ \text{b}\ \ \text{b}\ \ \text{b} \\ \hline\end{array}
\right)
=
\begin{array}{|l|}\hline w \\ \hline \ \ \ \ y\ \ x \\ \hline 0\ 1\ 2\ 3\ldots \\ \hline \text{b}\ \ \text{a}\ \ \text{b} \\ \text{b}\ \ \text{b}\ \ \text{b} \\ \hline\end{array}
\begin{array}{l} \\ \\ r \\ g_6 \\ g_8 \end{array}
$$

Q: Why so cumbersome? In particular: Why not have (12a), resulting in (12b)?

(12) a. $[\![\exists x]\!] = \bigcup_{d\in D} s[x/d]$
     b. $[\![\exists x\varphi]\!] = \left(\bigcup_{d\in D} s[x/d]\right)[\![\varphi]\!]$

A: For one purpose: To get the interpretation of sentences with $\Diamond$ right. Let's look at $\Diamond$.

- For any φ, $[\![\Diamond\varphi]\!]$ is a **test**:
  (13)
  $$s[\![\varphi]\!] = \begin{cases} s & \text{if } s \text{ can be updated with } \varphi \text{ without leading to the absurd empty state.} \\ \emptyset & \text{i.e., the absurd empty state, otherwise.} \end{cases}$$

This means that ‘$\Diamond$ *rain*’ cannot meaningfully be used to *convey* the information that it may be raining.

a. Either the agent already believes that it may be raining; then the update has no effect.
b. Or the agent already believes that it is not raining; then the update leads to inconsistency.

➠ This rule is a disappointment if you are looking for an account of how modalized sentences can convey new information. But that is not what this paper aims to accomplish. Here, modals are strictly epistemic and “self-referential” in the sense that they are always evaluated with respect to the information state of the evaluator, not some other agent.

Now back to $\exists$. Consider (14) and the two possible logical representations:

(14) There is a student who may come to my office today.
a. $\exists x[S(x) \wedge \Diamond C(x)]$
b. $\exists x S(x) \wedge \Diamond C(x)$

Let $D = \{a, b\}$; $W = \{w, v\}$; $w(S) = \{a, b\}$; $v(S) = \{b\}$; $w(C) = \{b\}$; $v(C) = \emptyset$. Notice that a does not come in either of the worlds. Start from the state of ignorance.

(15)
$$\bigcup_{d\in\{a,b,c\}}\left(\begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline & \\ \hline 0\ \ldots & 0\ \ldots \\ \hline & \\ \hline\end{array}[x/d][\![S(x)\wedge\Diamond C(x)]\!]\right)$$

$$=\bigcup\left(\begin{array}{l}\begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline \mathrm{x} & \mathrm{x} \\ \hline 0\ \ldots & 0\ \ldots \\ \hline \mathrm{a} & \mathrm{a} \\ \hline\end{array}[\![S(x)]\!][\![\Diamond C(x)]\!], \\ \\ \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline \mathrm{x} & \mathrm{x} \\ \hline 0\ \ldots & 0\ \ldots \\ \hline \mathrm{b} & \mathrm{b} \\ \hline\end{array}[\![S(x)]\!][\![\Diamond C(x)]\!]\end{array}\right) = \bigcup\left(\begin{array}{l}\begin{array}{|c|}\hline \mathbf{w} \\ \hline\hline \mathrm{x} \\ \hline 0\ \ldots \\ \hline \mathrm{a} \\ \hline\end{array}[\![\Diamond C(x)]\!], \\ \\ \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline \mathrm{x} & \mathrm{x} \\ \hline 0\ \ldots & 0\ \ldots \\ \hline \mathrm{b} & \mathrm{b} \\ \hline\end{array}[\![\Diamond C(x)]\!]\end{array}\right)$$

$$=\bigcup\left(\begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline \mathrm{x} & \mathrm{x} \\ \hline 0\ \ldots & 0\ \ldots \\ \hline \mathrm{b} & \mathrm{b} \\ \hline\end{array}\right) = \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v} \\ \hline\hline \mathrm{x} & \mathrm{x} \\ \hline 0\ \ldots & 0\ \ldots \\ \hline \mathrm{b} & \mathrm{b} \\ \hline\end{array}$$

This is how the sentence is interpreted according to GSV. The crucial feature of this rule is that the scope of $\exists x$ is processed “locally” for each possible value of x, not “globally” relative to the whole information state.

What would have happened if we had wrapped up the scope of $\exists x$ (i.e., taken the union) earlier?

(16) $$\bigcup_{d\in\{a,b,c\}}\left(\begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v}\\ \hline\hline & \\ \hline 0\ \ldots & 0\ \ldots \\ \hline & \\ \hline\end{array}[x/d][\![S(x)]\!]\right)[\![\Diamond C(x)]\!]$$

$$=\bigcup\left(\begin{array}{l} \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v}\\ \hline\hline \text{x} & \text{x}\\ \hline 0\ \ldots & 0\ \ldots\\ \hline \text{a} & \text{a}\\ \hline\end{array}[\![S(x)]\!], \\ \\ \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v}\\ \hline\hline \text{x} & \text{x}\\ \hline 0\ \ldots & 0\ \ldots\\ \hline \text{b} & \text{b}\\ \hline\end{array}[\![S(x)]\!]\end{array}\right)[\![\Diamond C(x)]\!] = \bigcup\left(\begin{array}{l} \begin{array}{|c|}\hline \mathbf{w}\\ \hline\hline \text{x}\\ \hline 0\ \ldots\\ \hline \text{a}\\ \hline\end{array} \\ \\ \begin{array}{|c|c|}\hline \mathbf{w} & \mathbf{v}\\ \hline\hline \text{x} & \text{x}\\ \hline 0\ \ldots & 0\ \ldots\\ \hline \text{b} & \text{b}\\ \hline\end{array}\end{array}\right)[\![\Diamond C(x)]\!]$$

$$=\begin{array}{|c|c|c|}\hline \mathbf{w} & \mathbf{w} & \mathbf{v}\\ \hline\hline \text{x} & \text{x} & \text{x}\\ \hline 0\ \ldots & 0\ \ldots & 0\ \ldots\\ \hline \text{a} & \text{b} & \text{b}\\ \hline\end{array}[\![\Diamond C(x)]\!] = \begin{array}{|c|c|c|}\hline \mathbf{w} & \mathbf{w} & \mathbf{v}\\ \hline\hline \text{x} & \text{x} & \text{x}\\ \hline 0\ \ldots & 0\ \ldots & 0\ \ldots\\ \hline \text{a} & \text{b} & \text{b}\\ \hline\end{array}$$

- This is a strange result: In this information state, it is believed that a is among possible referents of x for whom the sentence $S(x) \wedge \Diamond C(x)$ is true. But while a a student in his possibility, there is no possibility in which a comes!
- That did not happen in the earlier update in (15). There the second conjunct $\Diamond C(x)$ was evaluated "locally" for the set of possibilities that map x to d, for each individual $d \in D$.
- Intuitively, the difference between (15) and (16) in the interpretation of $\Diamond C(x)$ can be paraphrased as follows:

  (15') Among the possibilities in which x is assigned to d, there is one (or more) in which the referent of x comes.

  (16') There is one (or more) possibility in which the individual that x is assigned to comes.

  It is much easier for (16) to be true; but in our example, that is probably not the reading that we want. That's why GSV evaluate the scope of an existential quantifier "locally" for each individual. This doesn't quite make the problem go away, however. Consider the following:

  (17) a. There is a student who may come today. She may bring my book.
  b. There is a student who may come today and bring my book.

  At some point, we have to take the union and wrap up the existential quantifier. If we do this at the end of the sentence, then (17a) is not equivalent to (17b)!

- In general, the following equivalence was valid in Groenendijk and Stokhof (1991), but is not valid in Groenendijk et al. (1996):

$$\exists x\varphi \wedge \psi \not\Leftrightarrow \exists x[\varphi \wedge \psi]$$

since, as we saw, they are not equivalent if ψ contains a possibility operator.

- In Section 4.2, GSV argue that there are in fact examples where both readings are sensible.

(18) a. There is someone hidden in the closet. He might be guilty.
b. There is someone hidden in the closet who might be guilty.

I have to admit that this example is too subtle for me to be sure whether I agree.

Temporary states

In their subsection on **modal subordination**, GSV mention briefly discuss the potential use of the "temporary" states created in the course of various tests, esp. Negation and modals.

(19) A wolf might come in. It would eat you first.

They stop short of developing a fully account, but the idea is pretty clear:

i. The evaluation of $\Diamond\exists x[W(x)\wedge C(x)]$ in a state s consists in a test: Update s with $\exists x[W(x)\wedge C(x)]$ and observe what happens.

ii. If the test is successful, the state $s[\![\exists x[W(x)\wedge C(x)]$ is created. If we could keep it "in memory," we might be able to refer to it later, in processing the next sentence.

iii. Otherwise, the modal sentence is false and there is no use for $s[\![\exists x[W(x)\wedge C(x)]$.

Version of the GSV system are still very much being used. However, it can and must be flashed out in various ways. For instance, its account of '*might*' is only reasonable in self-referential epistemic uses (and even there it sounds like an odd depiction of what goes on in communication). The next paper will begin to do something about that.

3 Veltman (1996)

We are (for now) back to propositional logic; no idividuals. Veltman's goal in this paper is to develop an update semantics in which an information state is not just a representation of what is deemed **possible**, but also which worlds are more **likely** than others.

This is a rich and long paper, and it is impossible to go through it in detail in the time we have. I will try to isolate a few ideas that are particularly relevant.

Veltman's intended application is **default reasoning** – patterns like these:

(20) Ps normally are R.
x is P
―――――――――
Presumably, x is R.

(21) P's normally are R.
x is P.
x is Q.
―――――――――
Presumably, x is R.

(22) Q's normally are not R.
P's normally are R.
x is P.
x is Q.
―――――――――
???

(23) Q's normally are P.
Q's normally are not R.
P's normally are R.
x is P.
x is Q.
―――――――――
Presumably, x is not R.

Obviously, rules like (24a) are not the same as (24b), and (25a) is not the same as (25b).

(24) a. Q's normally are P
b. All Q's are P

(25) a. Presumably, x is not R.
b. x is not R.

In both (24) and (25), the (a)-sentence is a "soft belief" which the speaker realizes may have to be given up in light of new evidence, whereas the (b)-sentence is a "hard belief" which the speaker expects to be consistent with all new information.

➨ In other words, we are dealing with **non-monotonicity**.

We've seen this before: *might p* ; *not p* is consistent, whereas *not p* ; *might p* is not. For instance, the belief in of (26a) is given up as a result of the update with (26b), so that the continuation with (26c) sounds self-contradictory.

(26) a. Mary might be outside.
b. Mary is inside.
c. #Mary might be outside.

This already came out right in Groenendijk et al. (1996). Veltman wants to extend the account to modals like *'normally'* and *'presumably'*.

Might

- **Languages $\mathcal{L}^0_{\mathcal{A}}$, $\mathcal{L}^1_{\mathcal{A}}$:** Let a set of finitely many atomic sentences $\mathcal{A}$ be given.
  i. $\mathcal{L}^0_{\mathcal{A}}$ is the standard language of propositional logic built from $\mathcal{A}$.
  ii. $\varphi \in \mathcal{L}^0_{\mathcal{A}}$ iff $\varphi, might\ \varphi \in \mathcal{L}^1_{\mathcal{A}}$.
- **Information states:** Let W be the powerset of $\mathcal{A}$.
  i. σ is an information state iff $\sigma \subseteq W$;
  ii. $\mathbf{0}$, the minimal state, is the information state given by W;
  $\mathbf{1}$, the absurd state, is the information state given by the empty set.
  iii. For every two states σ and τ, $\sigma + \tau = \sigma \cap \tau$.[1]

  Note: This definition does not really tell us what a subset of $\mathcal{L}_{\mathcal{A}}A$ represents. Only the prose does. It's important to keep this in mind: "Intuitively, a subset w of $\mathcal{A}$ – or a possible world as we shall call it – will be an element of σ iff, for all the agent in σ knows, w might give a correct picture of the facts – given the agent's information, the possibility is not excluded that the atomic sentences in w are all true **and the others false**. (Emphasis added.)

  ➠ Although a world w is just a set of atomic sentences, implicitly the state of affairs depicted by w is taken to be one where all the sentences not in w are false.

  ➠ Worlds are **total**, as before.
- **Interpretation for $\mathcal{L}^1_{\mathcal{A}}$:** For every $\varphi \in \mathcal{L}^1_{\mathcal{A}}$ and state σ:

(27) $$\text{If } \varphi \in \mathcal{A}, \text{ then } \sigma[\![\varphi]\!] = \sigma \cap \{w \in W | p \in w\}$$

(28) $$\sigma[\![\neg\varphi]\!] = \sigma \setminus \sigma[\![\varphi]\!]$$

(29) $$\sigma[\![\varphi \wedge \psi]\!] = \sigma[\![\varphi]\!] \cap \sigma[\![\psi]\!]$$

(30) $$\sigma[\![\varphi \vee \psi]\!] = \sigma[\![\varphi]\!] \cup \sigma[\![\psi]\!]$$

(31) $$\sigma[\![might\ \varphi]\!] = \begin{cases} \sigma & \text{if } \sigma[\![\varphi]\!] \neq \mathbf{1} \\ \mathbf{1} & \text{if } \sigma[\![\varphi]\!] = \mathbf{1} \end{cases}$$

  Comments: This time, conjunction is not defined as sequential update – presumably because Veltman liked the correspondence with conjunction. Question: Is this definition of conjunction equivalent to one involving sequential update? Find out for yourself.

  'might' is again interpreted as a **test**.

[1] The + operator is defined as the join of the lattice of information states. I'd like to skip this part unless people want to discuss it.

- There are a lot more interesting points in Section 2, but we'll skip them.

Normally, presumably

- **Language $\mathcal{L}^2_{\mathcal{A}}$:** $\varphi \in \mathcal{L}^0_{\mathcal{A}}$ iff φ, *normally* φ, *presumably* $\varphi \in \mathcal{L}^2_{\mathcal{A}}$.
- Sentences with *'normally'* are called **(default) rules**
- For the interpretation of these new sentences, let's take an excursion to the land of modality. Although this is a bit of a disruption, it will provide the link to standard linguistic accounts of modality.

Excursion: Ordering sources

Ordering sources were introduced by Kratzer (1981). I've adapted the framework and notation here for Veltman's framework.

(32) a. This is the road to Springfield.
b. This must be the road to Springfield.

- *'must'* clearly "feels like" a necessity modal.
- But intuitively, (32b) is semantically *weaker* than (32a): You can believe (and assert) (32b) without believing (or asserting) (32a), but not *vice versa*.[2]
- Kratzer wants to treat *'must'* as a necessity modal *and* account for the fact that it gives rise to relatively weak readings. Ordering sources help her accomplish this.

Technicalities

i. Basic idea: Impose an *order* on the information state and let the quantification range only over the minimal elements of this order.

ii. Formally: The **proposition** expressed by φ is defined by Veltman as the set of worlds that you get when you update the "minimal state" **0** with φ:

$$||\varphi|| =_{df} \mathbf{0}[\![\varphi]\!]$$

iii. An order $\leqslant_\Phi$ on σ is derived from a set Φ of propositions "that a certain agent considers to be *normally* the case."

(33) $$w \leqslant_\Phi z \text{ iff } \{p|p \in \Phi \wedge z \in p\} \subseteq \{p|p \in \Phi \wedge w \in p\}$$

(Recall that a proposition is a set of worlds.)

- $\leqslant_\Phi$, for any Φ, is a *pre-order* (transitive and reflexive).
- This order does not give us "degrees" of conformity with the norms. We only get a way of *comparing* worlds and telling which is more/less exceptional; there is no absolute measure of "goodness."

[2]Kratzer's argument is slightly different: She notes that under a "'pure' epistemic interpretation (presumably a realistic one) of the modal, (32b) entails (32a). This is true, but the premise that (32b) has such a "'pure' epistemic reading is not so obvious to me.

Veltman

i. In his definitions, Veltman starts out with a given reflexive and transitive relation ε, only motivating it in his prose as being induced by a set of "normal" propositions in the way defined in (33). As a consequence, there is a slight notional difference: In '$w \leqslant_\varepsilon v$', the subscript is not the set of propositions, but the relation itself.

ii. According to Veltman, we should think of a statement of the form $w \leqslant_\varepsilon z$ as saying that w is **no more exceptional** than z. From this, we can also define an equivalence relation '$w \simeq_\varepsilon v$' as '$w \leqslant_\varepsilon v \wedge v \leqslant_\varepsilon w$', and ''$w <_\varepsilon v$', the statement that w is (strictly) less exceptional than v.

iii. If you look at Kratzer's (1981) truth conditions for modals (see my handout from the *Modality* tutorial in August, 2005), you see that it is quite complicated. That was because we cannot in general assume that there is a set of minimal $\leqslant_\varepsilon$-worlds. However, with Veltman we can: Since the set $\mathcal{A}$ is finite and possible worlds are subsets of $\mathcal{A}$, there are only finitely many possible worlds.

End of excursion

Continuing Veltman's definitions for $\mathcal{L}^2_{\mathcal{A}}$:

- **Exception pattern on W:** Let W be as before. Then ε is an **exception pattern** on W iff ε is a reflexive and transitive relation on W.
- Eventually, an information state will be defined as a pair $\langle \varepsilon, s\rangle$, where s is again a set of possible worlds and ε is an exception pattern. Some more auxiliary notions are required.
- **Normal worlds in W:** Let ε be an exception pattern on W.

    i. w is a **normal world** in ε iff $w \leqslant_\varepsilon v$ for every $v \in W$.

    ii. $\mathbf{n}\varepsilon$ is the set of all normal worlds in ε.

    iii. ε is **coherent** iff $\mathbf{n}\varepsilon \neq \emptyset$.

    Note: There are many transitive and reflexive relations under which there is no normal world according to this definition (even if the set of worlds is finite). In the present setup, we are only guaranteed to get normal wolds if the set of propositions generating ε is consistent.

- $\mathbf{n}\varepsilon$ is the set of "best" worlds relative to all of W. Now, and agent may know that she is not in one of those best worlds – none of them may be in her set of worlds s. However, ε can also give us a set best worlds among the ones in s. We simply restrict ε to s:
- **Optimal worlds in s:** Let ε be a pattern on W, and $s \subseteq W$.

    i. w is **optimal in** $\langle \varepsilon, s\rangle$ iff $w \in s$ and there is no $v \in s$ such that $v <_\varepsilon w$.

    ii. $\mathbf{m}_{\langle \varepsilon, s\rangle}$ is the set of all optimal worlds in $\langle \varepsilon, s\rangle$.

- **Refinement of an exception pattern:** Let $\varepsilon, \varepsilon'$ be patterns on W, and $e \subseteq W$.

    i. ε' is a **refinement** of ε iff $\varepsilon' \subseteq \varepsilon$;

ii. The **refinement of** ε **with** e is the exception pattern $\varepsilon \circ e$, defined as follows:

(34) $$w(\varepsilon \circ e)v \iff w\varepsilon v \wedge (v \in e \rightarrow w \in e)$$

That is, w is at least as good as w according to the original ranking, and if e holds in v, then it holds also in w.

This eliminates from ε those pairs of worlds $\langle w, v\rangle$ where v conforms to the new rule e and w doesn't.

If ε is generated by a set of propositions Φ (see Kratzer's system above), then this simply amounts to adding e to Φ.

- **Defaults:** A proposition $e \subseteq W$ is a **default in** ε iff $e \neq \emptyset$ and $\varepsilon \circ e = \varepsilon$.

  (I.e., if ϵ is not changed by refining it with e.)

- **Information states:** Let W as before.

  i. σ is an **information state** iff $\sigma = \langle \epsilon, s\rangle$ and one of the following is fulfilled:

     a. ε is a coherent pattern on W and s is a non-empty subset of W;

     b. $\varepsilon = \{\langle w, w\rangle | w \in W\}$ and $s = \emptyset$;

  ii. a. $\mathbf{0}$, the minimal state, is given by $\langle W \times W, W\rangle$;

     b. $\mathbf{1}$, the absurd state, is given by given by $\langle \{\langle w, w\rangle | w \in W\}, \emptyset\}$

  iii. Let $\sigma = \langle \varepsilon, s\rangle$ and $\sigma' = \langle \varepsilon', s'\rangle$ be states.

     a. $\sigma + \sigma' = \langle \varepsilon \cap \varepsilon', s \cap s'\rangle$ if $\langle \varepsilon \cap \varepsilon', s \cap s'\rangle$ is coherent;

     b. $\sigma + \sigma' = \mathbf{1}$ otherwise.

  The last clause ensures that only one incoherent state is ever reached: All updates that result in an incoherent state are "directed" to $\mathbf{1}$.

- **Update rules:** Let $\sigma = \langle \varepsilon, s\rangle$ be an information state. For all $\varphi \in \mathcal{L}^2_{\mathcal{A}}$:

(35) $$\text{if } \varphi \in \mathcal{L}^0_{\mathcal{A}}, \text{ then } \sigma[\![\varphi]\!] = \begin{cases} \mathbf{1} & \text{if } s \cap ||\varphi|| = \emptyset \\ \langle \epsilon, s \cap ||\varphi||\rangle & \text{otherwise} \end{cases}$$

(36) $$\text{if } \varphi = \textit{normally } \psi \text{ then } \sigma[\![\varphi]\!] = \begin{cases} \mathbf{1} & \text{if } \mathbf{n}\varepsilon \cap \psi = \emptyset \\ \langle \varepsilon \circ ||\psi||, s\rangle & \text{otherwise} \end{cases}$$

(37) $$\text{if } \varphi = \textit{presumably } \psi \text{ then } \sigma[\![\varphi]\!] = \begin{cases} \sigma & \text{if } \mathbf{m}_\sigma \cap ||\psi|| = \mathbf{m}_\sigma \\ \mathbf{1} & \text{otherwise} \end{cases}$$

- (35) updates factual knowledge only; (36) updates knowledge about defaults only; (37) is a test.

- (36) is new. We didn't have a way of learning about what's likely before.

- (37) is now a better test than what we had before: Only the set $\mathbf{m}_\sigma$ is tested, not all worlds in σ. Thus the agent may believe that ψ is *presumably* true without being sure that ψ is true.

- We can now also define a stronger version *may* φ: Not as 'φ *is possible*', but rather something like '*varphi is* presumably *possible*'. The latter imposes a stronger condition the set of worlds, namely that there be a φ-world in $\mathbf{m}_\sigma$, not just in σ.

Some consequences

The consequence relation $\Vdash$ is defined as follows:

(38) $$\psi_1, \ldots, \psi_n \Vdash \varphi \iff \mathbf{0}[\![\psi_1]\!] \ldots [\![\psi_n]\!] \Vdash \psi$$

(39) a. $\mathbf{0}[\![\textit{normally } p]\!][\![\neg p]\!] \neq 1$
$\mathbf{0}[\![\textit{normally } p]\!][\![\textit{normally } \neg p]\!] = 1$
b. $\textit{normally } p \Vdash \textit{presumably } p$
$\textit{normally } p, \neg p \nVdash \textit{presumably } p$
$\textit{normally } p, \neg p \Vdash \textit{normally } p$
c. $\textit{normally } p, q \Vdash \textit{presumably } p$
$\textit{normally } p, q, \neg p \nVdash \textit{presumably } p$
d. $\textit{normally } p, \textit{normally } q \Vdash \textit{presumably } p$
$\textit{normally } p, \textit{normally } q, \neg p \nVdash \textit{presumably } p$
$\textit{normally } p, \textit{normally } q, \neg p \Vdash \textit{presumably } q$
e. $\textit{normally } p, \textit{normally } q, \neg(p \wedge q) \nVdash \textit{presumably } p$
$\textit{normally } p, \textit{normally } q, \neg(p \wedge q) \nVdash \textit{presumably } q$

Conditional rules

In Section 4 of his paper, Veltman refines his system. We won't discuss this in as much detail as above, but it is important to know what he's doing there.

Consider (40):

(40) a. Normally it rains.
b. But if there is an easterly wind, the weather is usually dry.

The content of the "excpetion rule" (40b) cannot be captured adequately with a unary operator like *normally*. The best guess might be to put a material conditional in its scope:

(39') a. *normally rain*
b. *normally*(*eastwind* $\rightarrow \neg$ *rain*)

For the two statements in (40a,b) to be true, it is sufficient that there is normally no eastwind. Hence (40b) does not really impose the constraint that (39b) does.

Veltman's solution: Have *binary* operator $\rightsquigarrow$ to form **restricted rules**: $\varphi \rightsquigarrow \psi$ means intuitively '*If φ, then normally ψ*'.

➨ This is a statement about the φ-worlds in W.

NOT about the φ-worlds in the agent's information state, for one can say

(41) a. If there is an easterly wind, the weather is usually dry.
b. But today there is an easterly wind and it is raining.

So as before with the unary operator *normally*, it is possible to know that you are not in a normal world.

It seems that Veltman was not striving for originality when implementing the idea of conditional rules. He stipulates that the model fixes a "local" exception pattern for each subset of W.

Q: Why can't we derive these local patterns from the overall rleation ε?

A: I don't know. As far as I can see, it should be possible.

Upshot:

Richer formal representations of information states can model more than just an agent's beliefs of what is (im-)possible. If we enrich our notion of information state to model more subtle data, we will likely find that there is some linguistic expression that operates on the piece of the apparatus that incorporates the enrichment.
➨ There is no *a priori* limit on what you can do with dynamic semantics. But it can get complicated.

4 Roberts (1989)

Here are Roberts's examples of modal subordination:

(42) The birds will get hungry (this winter).

(43) a. If Edna forgets to fill the birdfeeder, she will feel very bad.
b. The birds will get hungry.

(44) a. If John bought a book, he'll be reading it by now.
b. #It's a murder mystery.

(45) a. If John bought a book, he'll be reading it by now.
b. It will be a murder mystery.

(46) a. A thief might break into the house.
b. He would take the silver.

Modal DRT

Roberts presents her account in an intensional version of DRT (see the end of yesterday's handout). Let's take a closer look at that.

i. The language of boxes is like on yesterday's handout, with the following additional forms:

(47) If $K_1, K_2 \in \kappa$, then $K_1 \Box K_2, K_1 \Diamond K_2 \in \kappa$.

ii. Let's not join Veltman in making too many simplifications. The set of worlds may be infinite, and there is not necessarily a set of "minimal" ones. Furthermore, the order $\leqslant$ depends on w via an *ordering source* g, a function from worlds to sets of propositions.

For any box K, let X_K be the set of discourse referents introduced in K. For any finite set $X = \{x_1, \dots, x_n\}$ of discourse referents, let $g[X]h \iff g[x_1] \circ \dots \circ [x_n]h$.

Definition (48) is not equivalent to Roberts's. I think this one is better, but I'll have to think more about it. No guarantee.

Definition (49) is a simpler version that works if we assume that there is a set of minimal or "best" worlds in s. Either way, $\Diamond$ is the converse of $\Box$.

$$(48)\qquad g[\![K_1 \Box K_2]\!]_w h \iff h = g \wedge \\ \forall i, v[(h[X_{K_1}]i \wedge v \in s \wedge i[\![K_1]\!]_v i) \rightarrow \\ \exists j, y[h[X_{K_1}]j \wedge y \in s \wedge y \leqslant_{g(w)} v \wedge j[\![K_1]\!]_y j \wedge \\ \forall k, z[(h[X_{K_1}]k \wedge z \in s \wedge z \leqslant_{g(w)} y \wedge k[\![K_1]\!]_z k) \rightarrow \\ \exists l[k[X_{K_2}]l \wedge l[\![K_2]\!]_z l]]]]$$

$$(49)\qquad g[\![K_1 \Box K_2]\!]_w h \iff h = g \wedge \\ \forall i, v[(h[X_{K_1}]i \wedge v \in \mathbf{m}_{\langle g(w),s\rangle} \wedge i[\![K_1]\!]_v i) \rightarrow \exists j[i[X_{K_2}]j \wedge j[\![K_2]\!]_v j]]$$

$$(50)\qquad K_1 \Diamond K_2 =_{df} \neg(K_1 \Box \neg K_2)$$

iii. Sentences like *might* p can be represented as conditional structures $K_1 \Diamond K_2$, where K_1 is empty i.e., imposes a trivial restriction and K_2 represents p.

Notice Roberts' use of the connective '□', rather than '⇒', for conditionals, highlighting the fact that conditionals are essentially modal expressions.

(45a')

$$\begin{array}{|c|}\hline \begin{array}{|c|}\hline x\ y \\ \hline \text{John(x)} \\ \text{book(y)} \\ \text{bought(x,y)} \\ \hline \end{array} \; \Box \; \begin{array}{|c|}\hline z\ w \\ \hline \text{reading(z,w)} \\ \text{z = x} \\ \text{w = y} \\ \hline \end{array} \\ \hline \end{array}$$

Now there are two ways to continue the update:

(45a' + 45b')

$$\begin{array}{|c|}\hline r \\ \hline \begin{array}{|c|}\hline x\ y \\ \hline \text{John(x)} \\ \text{book(y)} \\ \text{bought(x,y)} \\ \hline \end{array} \; \Box \; \begin{array}{|c|}\hline z\ w \\ \hline \text{reading(z,w)} \\ \text{z = x} \\ \text{w = y} \\ \hline \end{array} \\ \text{murder-mystery(r)} \\ \hline \end{array}$$

(45a' + 45b")

$$\begin{array}{|c|}\hline \begin{array}{|c|}\hline x\ y \\ \hline \text{John(x)} \\ \text{book(y)} \\ \text{bought(x,y)} \\ \hline \end{array} \; \Box \; \begin{array}{|c|}\hline z\ w\ r \\ \hline \text{reading(z,w)} \\ \text{z = x} \\ \text{w = y} \\ \text{murder-mystery(r)} \\ \text{r = y} \\ \hline \end{array} \\ \hline \end{array}$$

Clearly in (45a' + 45b'), there is no way to ensure that r refers to the book John bought in the embedded DRS. That information is lost once the processing of the conditional is over. Instead, something like (45a' + 45b") gives intuitively the right truth conditions.

Hypothesis 1: Insertion

Stick the new material into the embedded box of the preceding sentence.

Works for conditionals (see above)

BUT doesn't work for others: In (51b), the resulting meaning is '*A thief might break into the house and steal the silver*'.

(51) a. A thief might break into the house.

$$\boxed{\Diamond \boxed{\begin{array}{c} x \\ \hline \text{thief(x)} \\ \text{break-in(x)} \end{array}}}$$

b. He would take the silver.

$$\boxed{\Diamond \boxed{\begin{array}{c} x \\ \hline \text{thief(x)} \\ \text{break-in(x)} \\ \text{take-silver(x)} \end{array}}}$$

Hypothesis 2: Accommodation of the missing antecedent

Roberts instead argues for a purely pragmatic account: Accommodation (familiar from Lewis's writings on presupposition).

(52) a. A thief might break into the house.

$$\boxed{\Diamond \boxed{\begin{array}{c} x \\ \hline \text{thief(x)} \\ \text{break-in(x)} \end{array}}}$$

b. He would take the silver.

$$\boxed{\begin{array}{c} \Diamond \boxed{\begin{array}{c} x \\ \hline \text{thief(x)} \\ \text{break-in(x)} \end{array}} \\ \boxed{\begin{array}{c} x \\ \hline \text{thief(x)} \\ \text{break-in(x)} \end{array}} \; \Box \; \boxed{\begin{array}{c} y \\ \hline \text{take-silver(y)} \\ \text{y = x} \end{array}} \end{array}}$$

➠ Technically, Roberts proposes a kind of copy-and-paste procedure. This is of a very general kind and also works for conditionals:

(53)

$$\boxed{\begin{array}{c} \boxed{\begin{array}{c} x\ y \\ \hline \text{John(x)} \\ \text{book(y)} \\ \text{bought(x,y)} \end{array}} \; \Box \; \boxed{\begin{array}{c} z\ w \\ \hline \text{readingz,w} \\ \text{z = x} \\ \text{w = y} \end{array}} \\ \boxed{\begin{array}{c} x\ y \\ \hline \text{John(x)} \\ \text{book(y)} \\ \text{bought(x,y)} \end{array}} \; \Box \; \boxed{\begin{array}{c} r \\ \hline \text{murder-mystery(r)} \\ \text{r = y} \end{array}} \end{array}}$$

(54) Either there's no bathroom in this house or it's in a funny place.

$$\boxed{\boxed{\neg \boxed{\begin{array}{c} x \\ \hline \text{bathroom(x)} \\ \text{in-this-house(x)} \end{array}}} \; \vee \; \boxed{\boxed{\begin{array}{c} x \\ \hline \text{bathroom(x)} \\ \text{in-this-house(x)} \end{array}} \; \Box \; \boxed{\begin{array}{c} y \\ \hline \text{funny-place(y)} \\ \text{y = x} \end{array}}}}$$

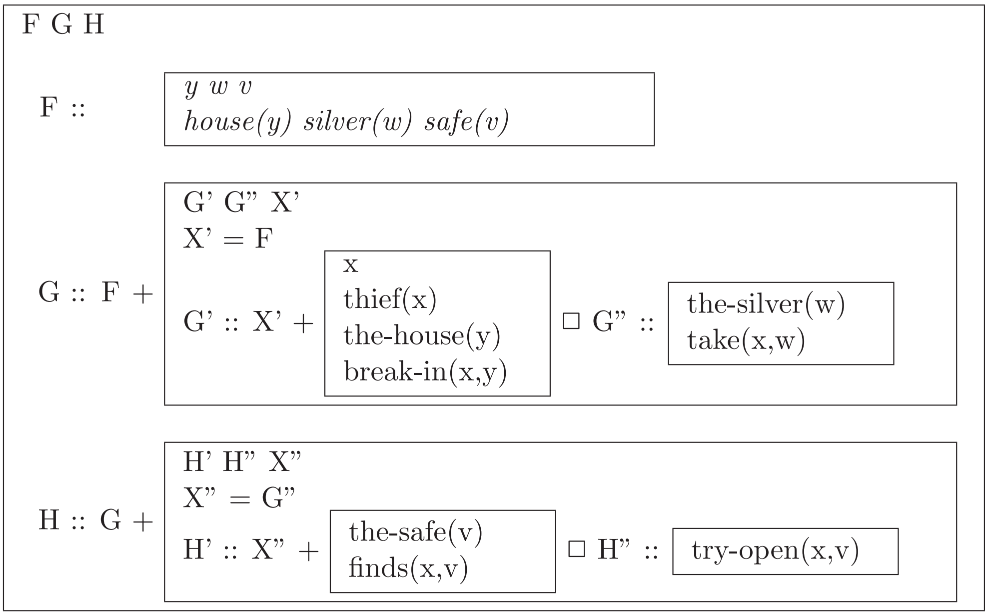

Figure 1: Frank-style Labeled DRS

Upshot

Roberts finds a way to express the meaning of the sequences in boxes, but there is no systematic way to explain when, why and exactly how accommodation takes place. Instead of the copy-and-paste method, some subsequent authors have used cross-references within the larger box (e.g., Frank’s 1996 “labeled” Discourse Representation Theory (LDRT); see Figure 1 for illustration.) We will leave this subject here and turn to a different method.

5 Kaufmann (2000)

One of the problems with Roberts’s account, which is shared by those that use anaphoric links between boxes, is that the phenomenon is more “local” than those methods would suggest:

I will be staying within a propositional framework here.

i. The formalism is based upon Groenendijk et al. (1996).

ii. Knowing in one state about another state:

(55) a. $s[t \vdash \psi] = \{i \in s |\ i \not\in t \text{ or } i \in t[\psi]\}$
b. $s[t \vdash \psi] = s[\varphi_{s,t} \rightarrow \psi]$

Here $\varphi_{s,t}$ is any proposition that leads from s to t. (There are of course many.)

iii. Failure

(56) a. $s[\perp] = \emptyset$
b. $t \vdash \perp \Leftrightarrow t = \emptyset$
c. $s[t \vdash \perp] = \{i \in s | i \not\in t \text{ or } i \in t[\perp]\}$
$= s[\neg\varphi_{s,t}]$

Thus given the obvious consequence in (56b), we can define negation as implication as in (56c).

iv. Success

(57) a. $t \nvdash \bot \Leftrightarrow t \neq \emptyset$
b. $s[t \nvdash \bot] = \{i \in s | \exists i' \in s. i' \in t\}$
$= s[\Diamond \varphi_{s,t}]$
c. $[t \vdash \top] \overset{def}{\equiv} [t \nvdash \bot]$

There is still a sense in which the definitions in (56) and (57) are fundamentally different: The former can actually add information to the input state by eliminating some, but not all possibilities. The latter can only return the input itself upon success or the empty state, indicating failure. These different behaviors are a consequence of the set-up of the current system: Since the only operation on states is elimination of possibilities, a statement that t is empty can be made true by eliminating all possibilities that are also in t. A statement that t is not empty, on the other hand, cannot be made true if it is false, since there is no way of "putting back" possibilities.

v. Stacks

Definition 1 (Stacks)
The set of stacks is defined as follows:

a. *If s is an information state, then $\langle s \rangle$ is a stack.*
b. *If s is an information state and σ is a stack, then $\langle s, \sigma \rangle$ is a stack.*
c. *Nothing else is a stack.*

Stacks are used here with the assumption that the bottom state represents the indicative, "original" state, while all others are "temporary." An empty stack is not defined, nor can the operations below produce one, since all interpretation by definition takes place in some state.

Definition 1 (Stack Operations) *The set of stack operators includes the following:*

a. Assume: $\langle s, \sigma \rangle \, [\varphi]^{\uparrow} \, \langle s[\varphi], \langle s, \sigma \rangle \rangle$

b. Conclude: $\langle s, \sigma \rangle \, [\varphi]_{\downarrow} \, \langle s[\varphi], \tau \rangle$,
where $|\sigma| = |\tau| = n$
and $\sigma_i [s \vdash \varphi] \tau_i$ for all $i,\ 0 \leqslant i < n$

c. Pop: $\langle s, \sigma \rangle \, [\nabla] = \begin{cases} \langle s \rangle \text{ if } \sigma = \langle \rangle \\ \sigma \text{ otherwise} \end{cases}$

vi. Illustration

(58) a. $s[\varphi \rightarrow \psi]$
b. $s \, [\varphi]^{\uparrow} \, [\psi]_{\downarrow}$

(59) $\left\langle \; s \; \right\rangle \; [\varphi]^{\uparrow} \left\langle \begin{array}{c} s[\varphi] \\ s \end{array} \right\rangle \; [\psi]_{\downarrow} \left\langle \begin{array}{c} s[\varphi][\psi] \\ s[s[\varphi] \vdash \psi] \end{array} \right\rangle$

Now suppose, for instance, that the next update is $[\chi]_\downarrow$. Let us abbreviate $s[s[\varphi] \vdash \psi]$ as s' and $s[\varphi][\psi]$ as t'. The stack undergoes the changes shown in (60):

(60) $\left\langle \begin{matrix} t' \\ s' \end{matrix} \right\rangle [\chi]_\downarrow \left\langle \begin{matrix} t'[\chi] \\ s'[t' \vdash \chi] \end{matrix} \right\rangle$

The operation that takes s' to $s'[t' \vdash \chi]$ is the removal of all those possibilties that have descendants in t' but not in $t'[\chi]$. But since $t' = s'[\varphi][\psi]$, this is equivalent to removing all those possibilities that have descendants in $s'[\varphi][\psi] = s'[\varphi;\psi]$, but not in $s'[\varphi][\psi][\chi] = s'[\varphi;\psi][\chi]$. From this and the definition of implication it is obvious that the end result is the equivalent of obtaining $s[t' \vdash \chi]$ by an update with $[(\varphi;\psi) \to \chi]$.

vii. Concluding $\top$ and $\bot$

(61) Conclude2: $\langle t, \langle s, \sigma \rangle\rangle\, [\varphi]_\Downarrow\, \langle t, \langle s[\varphi], \tau\rangle\rangle$,
where $|\sigma| = |\tau| = n$
and $\sigma_i[s \vdash \varphi]\tau_i$ for all $i,\ 0 \leqslant i < n$

Example:

(62) $\langle\ s\ \rangle\ [\varphi]^\uparrow \left\langle \begin{matrix} s[\varphi] \\ s \end{matrix} \right\rangle [\top]_\Downarrow \left\langle \begin{matrix} s[\varphi] \\ s[s[\varphi] \vdash \top] \end{matrix} \right\rangle$

viii. Denotations

Definition 2 (Translations) *A translation* $[\![\cdot]\!]$ *maps linguistic expressions to stack operations:*

a. $[\![\varphi]\!] = [\nabla]^\star\, [\varphi]_\downarrow$
b. $[\![\text{anyway}]\!] = [\![\text{however}]\!] = \ldots = [\nabla]^+$
c. $[\![\text{then } \varphi]\!] = [\![{}^{\mathbf{w}}\varphi]\!] = [\varphi]_\downarrow$
d. $[\![\text{if } \varphi]\!] = [\varphi]^\uparrow$
e. $[\![\text{not } \varphi]\!] = [\varphi]^\uparrow\, [\bot]_\Downarrow$
f. $[\![\text{might } \varphi]\!] = [\varphi]^\uparrow\, [\top]_\Downarrow$

ix. Illustration

(63) a. If Edna forgets to fill the birdfeeder, she will feel very bad.
b. The birds will get hungry.
c. if φ then ψ ; ${}^{\mathbf{w}}\chi$
d. $[\varphi]^\uparrow$; $[\psi]_\downarrow$; $[\chi]_\downarrow$

(64) a. $\langle\ s\ \rangle\ [\varphi]^\uparrow \left\langle \begin{matrix} s[\varphi] \\ s \end{matrix} \right\rangle [\psi]_\downarrow \left\langle \begin{matrix} s[\varphi][\psi] \\ s[s[\varphi] \vdash \psi] \end{matrix} \right\rangle$

b. $\left\langle \begin{matrix} s[\varphi][\psi] \\ s[s[\varphi] \vdash \psi] \end{matrix} \right\rangle [\chi]_\downarrow \left\langle \begin{matrix} s[\varphi][\psi][\chi] \\ s[s[\varphi] \vdash \psi][s[\varphi][\psi] \vdash \chi] \end{matrix} \right\rangle$

(65) a. A thief might come in.
b. He would take the silver.
c. might φ ; ${}^{\mathbf{w}}\psi$
d. $[\varphi]^\uparrow$; $[\top]_\Downarrow$; $[\psi]_\downarrow$

(66) a. $\langle\ s\ \rangle\ [\varphi]^{\uparrow} \left\langle \begin{array}{c} s[\varphi] \\ s \end{array} \right\rangle [\top]_{\Downarrow} \left\langle \begin{array}{c} s[\varphi] \\ s[s[\varphi] \vdash \top] \end{array} \right\rangle$

b. $\left\langle \begin{array}{c} s[\varphi] \\ s[s[\varphi] \vdash \top] \end{array} \right\rangle [\psi]_{\downarrow} \left\langle \begin{array}{c} s[\varphi][\psi] \\ s[s[\varphi] \vdash \top][s[\varphi] \vdash \psi] \end{array} \right\rangle$

x. Disjunction: Exclusive *'or'*

(67) a. You will stay unmarried, or you will marry a tramp.
b. You'll become a nun, or the tramp will beat you regularly.
c. Either way you'll have a miserable life.

(68) a. $s[\varphi \text{ xor } \psi] = \langle s[\varphi], s[\psi] \rangle$
b. $\langle s, t\rangle[\varphi \text{ xor } \psi] = \langle s[\varphi], t[\psi] \rangle$
c. $[\varphi \text{ xor } \psi]_{\downarrow} \equiv [\varphi]_{\downarrow} \text{ xor } [\psi]_{\downarrow}$

For "bathroom" sentences, assuming the equivalence in (68c) and assuming that the interpreter knows how to make the choice between the operators $[\cdot]_{\downarrow}$ and $[\cdot]_{\Downarrow}$, one would obtain (69):

(69) $\langle\ s\ \rangle\ [\varphi]^{\uparrow} \left\langle \begin{array}{c} s[\varphi] \\ s \end{array} \right\rangle [\bot \text{ xor } \psi]_{\downarrow} \left\langle \begin{array}{c} s[\varphi] \\ \langle s[s[\varphi] \vdash \bot], s[s[\varphi] \vdash \psi] \rangle \end{array} \right\rangle$

To be sure, although such an alteration would work for the examples above, it might turn out that as it stands it would be no less stipulative than previous proposals. It remains to be seen whether the framework proposed here can offer a substantial improvement in these cases.

6 Isaacs and Rawlins (to appear)

I expect that there will be little if any time to talk about this paper. That's a pity because it is a very nice one. Some data:

(70) If Alfonso comes to the party, will Joanna leave?

Some issues:

- How can we characterize the meaning of such sentences?

  Clearly this has to involve the semantics of questions; however, there have so far been no good ways to build compounds of sentences with questions in them. (But see recent work by Groenendijk, to appear, 2008.)

- Usually questions are characterized in terms of their answers (either the possible ones or the true ones, depending on who you ask). What are the possible answers to conditional questions? In particular, what's the status of (71) in response to (70)?

  (71) Alfonso isn't coming.

(72) a. A thief might break in. Do you think he would take the silver?
b. Yes, he would.

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