1 Introduction

1.1 What we will be talking about

Some eloquent expressions of very fundamental ideas motivating the dynamic turn:

• Karttunen (1969):

Consider a device intended to read a text in some natural language, interpret it, and store the content in some manner, say, for the purpose of being able to answer questions about it. To accomplish this task, the machine will have to fulfill at least the following basic requirement. It has to be able to build a file that consists of records of all the individuals, that is, events, objects, etc., mentioned in the text and, for each individual, record whatever is said about it. . . . In this paper, I intend to discuss one particular feature a text interpreter must have: that it must be able to recognize when a novel individual is mentioned in the input text and to store it along with its characteristics for future reference.

• Stalnaker (1978):

Let me begin with some truisms about assertions. First, assertions have content; an act of assertion is, among other things, the expression of a proposition — something that represents the world as being a certain way. Second, assertions are made in a context — a situation that includes a speaker with certain beliefs and intentions, and some people with their own beliefs and intentions to whom the assertion is addressed. Third, sometimes the content of the assertion is dependent on the context in which it is made, for example, on who is speaking or when the assertion takes place. Fourth, acts of assertion affect, and are intended to affect, the context, in particular the attitudes of the participants in the situation; how the assertion affects the context will depend on its content.

• Kamp (1981)

Two conceptions of meaning have dominated formal semantics of natural language. The first of these sees meaning principally as that which determines conditions of truth. This notion, whose advocates are found mostly among philosophers and logicians, has inspired the disciplines of truth-theoretic and model-theoretic semantics. According to the second conception meaning is,
first and foremost, that which a language user grasps when he understands the words he hears or reads. This second conception in many studies by computer scientists (especially those involved with artificial intelligence), psychologists and linguists – studies which have been concerned with to articulate the structure of the representations which speakers construct in response to verbal inputs.

1.2 Some observations

The scope of indefinites

Traditionally in logic, indefinite noun phrases like ‘a boy’ have been treated as existential quantifiers:

(1) a. A boy met a girl. \(\exists x[B(x) \land \exists y[G(y) \land M(x, y)]]\)

But whereas the existential quantifier must have a definite scope, the variables introduced by indefinite NPs are available for further reference indefinitely:

(2) a. A boy met a girl. \(\exists x[B(x) \land \exists y[G(y) \land M(x, y)]]\)
   b. She smiled. \(S(y)\)

The continuation in (2b) is fine in English, with the pronoun picking up the girl as its referent. But there is no way in traditional systems to get the formula \(S(y)\) into the scope of \(\exists y\).

\[\Rightarrow\text{The scope of indefinite noun phrases extends beyond the sentence to the right.}\]

Order matters

Discourse referents can only be referred to anaphorically after they have been introduced.

(3) a. A man\(_1\) walks in the park. He\(_1\) whistles.
   b. #He\(_1\) whistles. A man\(_1\) walks in the park.

The sequence in (3b) is well-formed, but the pronoun cannot have the intended referent.

\[\Rightarrow\text{The scope of indefinite noun phrases does not extend to the left.}\]

Specificity

This is a note on what kind of data we are not looking at.

With indefinite NPs, the accessibility for anaphoric reference in subsequent discourse often depends on specificity.\(^1\)

(4) Bill didn’t see a misprint.
   a. There is a misprint which Bill didn’t see.
   b. Bill saw no misprint.

\(^1\)Specificity is often equated with wide scope, but that is not the correct generalization, for reasons that we can’t get into here.
Both (4a) and (4b) are good paraphrases of the sentence, although it has just one of the two on any particular occasion of use. The two differ in their behavior with regard to anaphoric accessibility:

(5) a. There is a misprint which Bill didn’t see. It is on page ten.
    b. #Bill saw no misprint. It is on page ten.

In discussions of anaphoric accessibility, usually the non-specific reading of indefinites is intended unless indicated otherwise. This is because the behavior of the specific reading is much more predictable (and boring).

Karttunen explores the anaphoric accessibility of discourse referents introduced in various kinds of embedding context. He uncovers a wealth of interesting patterns, not all of which are fully understood to this day. The following are some examples.

**Conditionals**

The problem with “donkey sentence” is that it is impossible to give them a plausible logical form in which the indefinites are represented by existential quantifiers.

(6) If a farmer \(i\) owns a donkey \(j\), he \(i\) beats it \(j\).

- a. \(X \exists x [F(x) \land \exists y[D(y) \land O(x,y)]] \rightarrow B(x,y)\) \hspace{1cm} \((x, y \text{ unbound})\)
- b. \(X \exists x \exists y[F(x) \land D(y) \land \land O(x,y)] \rightarrow B(x,y)\) \hspace{1cm} \((x, y \text{ unbound})\)
- c. \(X \exists x \exists y[(F(x) \land D(y) \land O(x,y))] \rightarrow B(x,y)\) \hspace{1cm} \((\text{specific})\)
- d. \(\forall x \forall y[F(x) \land D(y) \land O(x,y)] \rightarrow B(x,y)\)

- In (6a,b), the occurrences of \(x,y\) to the right of the arrow are not bound by the quantifier. Technically, this is ok, but it means that the expressions to not properly mirror the meaning of the English sentence.
  - This is another example of the need to relax the right boundary of the scope of existential quantifiers (see above).
- In (6c), all the variables are bound, but the meaning again does not reflect that of the English sentence.
  - If we could somehow extend the scope of existentials to the right, that alone doesn’t guarantee that we would get the right reading.
- (6d) is fine, but it involves universal quantification. Moreover, its structure does not (even remotely) match that of the English sentence.
  - Getting the readings right would involve stipulating an ambiguity of indefinites between existential and universal quantification, plus a host of unsystematic structural stipulations.

Referents introduced in the antecedent of a conditional are available for reference in the consequent, but not in subsequent discourse:

(7) a. If a farmer \(i\) owns a donkey \(j\), he \(i\) beats it \(j\) because he \(i\) doesn’t like its \(j\) attitude.
    b. If a farmer \(i\) owns a donkey \(j\), he \(i\) beats it \(j\). #He \(i\) doesn’t like its \(j\) attitude.

The scope of indefinites should be open to the right within conditionals as well. However, it it should be restricted to the “local” context, as it cannot extend to subsequent sentences.
Negation

Anaphoric expressions in subsequent sentences usually cannot co-refer with indefinites introduced in the scope of negation.

(8) a. Bill has a car.  
    b. It is black.  
    c. The car is black.  
    d. Bill’s car is black.

(9) a. Bill doesn’t have a car.  
    b. #It is black.  
    c. #The car is black.  
    d. #Bill’s car is black.

This phenomenon has nothing to do with the existence or non-existence of the actual car. In the following, the same pattern arises even though we know that unicorns don’t exist.

(10) a. Bill saw a unicorn.  
    b. The unicorn had a gold mane.

(11) a. Bill didn’t see a unicorn.  
    b. #The unicorn had a gold mane.

➽ Negation usually isolates indefinites in its scope from anaphoric reference by items outside of its scope.

Q: Why usually?
A: Because there are exceptions:

(12) a. Bill doesn’t have a car.  
    b. #It is parked outside.  
    c. ✓ It would be parked outside.

➽ The barrier erected by negation can be overcome by anaphoric expressions occurring in certain modal contexts (more below).

Suppositions and modality

The following are due to Karttunen (1969).

(13) a. You must write a letter to your parents. It has to be sent by airmail. The letter must get there by tomorrow.  
    b. You must write a letter to your parents. #They are expecting the letter.

(14) a. Suppose Mary had a car. She takes me to work in it. I can drive the car too.  
    b. If Mary has a car, she will take me to work in it. I can drive the car too.  
    c. If Mary had a car, she would take me to work in it. I could drive the car too.  
    d. I wish Mary had a car. She could take me to work in it. I could drive the car too.  
    e. When Mary has a car, she can take me to work in it. I can drive the car too.

“All of the above examples elaborate a hypothetical situation that is based on the counterfactual or dubious premise that Mary has a car.” (Karttunen)

This pretty much sums up the idea behind much subsequent work on modal subordination, which we will discuss tomorrow.

Notice also that in many cases of modal subordination, the overall discourse gets a “conditional” reading:
A thief might come in. He would steal the silver.

Karttunen also noticed that a certain congruence must hold between the sentences:

a. I wish Mary had a car. #I will drive it.

b. If Mary has a car, she will take me to work in it. It {is / will be / would be} a Mustang.

c. If Mary had a car, she would take me to work in it. It {is / will be / would be} a Mustang.

Although modals usually block anaphoric coreference, this blockade can be overcome if the anaphoric expressions is embedded in a similar modal context.

**Attitude descriptions**

Sometimes anaphoric pronouns in subsequent discourse can disambiguate earlier indefinites:

a. Mary wants to marry a rich man. He is a banker.
   ➢ only specific

b. Mary wants to marry a rich man. He must be a banker.
   ➢ non-specific with deontic ‘must’; specific with epistemic ‘must’.

This sort of thing can even happen across belief holders. In (18a), the co-reference is ok even on a **non-specific** reading.

a. John believes that a squirrel ate his breakfast, and Mary believes that it ruined the flowerbed.

b. Bill hopes that a tornado will destroy the school building, and Freddy hopes it will level the police station, too.

Cross-sentential coreference is felicitous between similar attitude descriptions, even across different attitude holders.

### 1.3 The plan

Today, we will focus on reference to individuals. We will study three major semantic frameworks that have been proposed in this domain. The goal is to acquire some familiarity with these formalisms, to get an intuitive idea of the essential ideas behind them, ultimately to be able to express our own ideas and hypotheses in terms of these frameworks. Since we have only a few hours, we will be selective in the topics we cover. We won’t be superficial, however. The topics we do cover, we will look closely.
2 Preliminaries

We will be talking about variable assignments a lot. Just to brush up on some formal basics, here are the definitions of a typical “static” logical language and model-theoretic interpretation.

Language $L_A$: 

i. $V = \{x_1, x_2, \ldots, y, \ldots\}$ is a set of individual variables

ii. $C$ is a set of individual constants $a, b, \ldots$ and $n$-ary predicate constants, $1 \leq n$.

iii. $T = V \cup C$ is the set of terms, written $t_1, t_2, \ldots$ below.

iv. If $t_1, \ldots, t_n \in T$ and $P$ is an $n$-ary predicate constant, then $P(t_1, \ldots, t_n) \in L_A$.

v. If $t_1, t_2 \in T$, then $t_1 = t_2$ in $L_A$.

vi. If $\varphi, \psi \in L_A$, then $\neg \varphi, \varphi \land \psi, \varphi \lor \psi, \varphi \rightarrow \psi \in L_A$.

vii. If $x \in V$ and $\varphi \in L_A$, then $\exists x \varphi, \forall x \varphi \in L_A$.

viii. Nothing else is in $L_A$.

Model for $L_A$: A model is a pair $M = (D, I)$, where $D$ is non-empty set of individuals (the “domain”) and $I$ maps individual variables to elements of $D$ and $n$-ary predicate constants to sets of $n$-tuples of elements of $D$.

Variable assignment: A variable assignment is a function $g : V \mapsto D$.

Alternative assignment: This is merely a notational convenience which we will encounter below. For each variable $x \in V$, let $[x]$ be a relation between variable assignments such that $g[x|h] \iff$ for all variables $y \neq x, g(y) = h(y)$.

Semantics for $L_A$: A function $\llbracket \cdot \rrbracket^M_g$ for $L_A$ relative to model $M$ and assignment $g$ is defined as follows:

$$\llbracket t \rrbracket^M_g = \begin{cases} I(t) & \text{if } t \in C \\ g(t) & \text{if } t \in V \end{cases}$$

$$\llbracket \varphi \rrbracket^M_g \in \{0, 1\},$$

as follows:

$$\llbracket t_1 = t_2 \rrbracket^M_g = 1 \iff \llbracket t_1 \rrbracket^M_g = \llbracket t_2 \rrbracket^M_g$$

$$\llbracket P(t_1, \ldots, t_n) \rrbracket^M_g = 1 \iff \llbracket P(t_1), \ldots, \llbracket t_2 \rrbracket^M_g \rrbracket \in I(P)$$

$$\llbracket \neg \varphi \rrbracket^M_g = 1 \iff \llbracket \varphi \rrbracket^M_g = 0$$

$$\llbracket \varphi \land \psi \rrbracket^M_g = 1 \iff \llbracket \varphi \rrbracket^M_g = 1 \text{ and } \llbracket \psi \rrbracket^M_g = 1$$

$$\llbracket \varphi \lor \psi \rrbracket^M_g = 1 \iff \llbracket \varphi \rrbracket^M_g = 1 \text{ or } \llbracket \psi \rrbracket^M_g = 1$$

$$\llbracket \varphi \rightarrow \psi \rrbracket^M_g = 1 \iff \llbracket \varphi \rrbracket^M_g = 0 \text{ or } \llbracket \psi \rrbracket^M_g = 1$$

$$\llbracket \exists x \varphi \rrbracket^M_g = 1 \iff \text{for some } h \text{ such that } g[x|h], \llbracket \varphi \rrbracket^{M,h} = 1$$

$$\llbracket \forall x \varphi \rrbracket^M_g = 1 \iff \text{for all } h \text{ such that } g[x|h], \llbracket \varphi \rrbracket^{M,h} = 1$$

The last two clauses manipulate variable assignments. In a certain sense, they can be read as modal statements: The actual $g$ does not matter; rather, these statements quantify over possible variable assignments:

(19) a. $\exists x \varphi$ is true in $M, g$ iff $\varphi$ can be true in $M$ for some alternative assignment.

b. $\forall x \varphi$ is true in $M, g$ iff $\varphi$ must be true in $M$ for any alternative assignment.

In general in dynamic semantics, the trick is to keep all of these “alternative assignments” in memory, so to speak.

Exactly which alternatives are kept in memory depends on previously acquired information about the referent of $x$. If it is accepted that $g(x)$ is a man, then alternatives which don’t map $x$ to a man are discarded.
3 Heim (1983b)

Note: Heim’s paper is mostly about presupposition projection, but that’s not why we are discussing it. It’s true, though, that presupposition projection is one of the areas in which dynamic semantics has had a deep and lasting impact. I chose this paper because it is a very concise and easy-to-read exposition of some basic ideas. But since it is about presuppositions, we’ll take a brief look at that as well.

3.1 Some motivating examples

(20) a. John’s daughter is bald. 
   \( \Rightarrow \) John has a daughter.

b. John has a daughter and his daughter is bald. 
   \( \not\Rightarrow \) John has a daughter.

c. #John’s daughter is bald and he has a daughter. 
   \( \Rightarrow \) John has a daughter.

d. If John has a daughter, his daughter is bald. 
   \( \not\Rightarrow \) John has a daughter.

e. If John has a son, his daughter is bald. 
   \( \Rightarrow \) John has a daughter.

f. #If John’s daughter is bald, he has a daughter.
   \( \Rightarrow \) John has a daughter.

g. John doesn’t have a daughter. #She is bald.

Whether a presupposition that is triggered somewhere in a sentence (here by the possessive) becomes a presupposition of the whole sentence (i.e., projects) depends on both the structure and the content of the whole sentence (not just the trigger).

- Asymmetry of conjunction and conditionals

- Presupposition doesn’t project if it is entailed by an “earlier” part of the sentence (left conjunct, antecedent)

3.2 Main ideas

- Sentences are interpreted (or “processed”) in stepwise fashion, one piece at a time, usually left-to-right.

- Asymmetries

- Sentences are interpreted in a context, and their interpretation in turn affects that context (cf. Stalnaker, 1978).

- Hence the meanings of sentences are called context change potentials (CCP).

- More simply, the idea is that sentences denote relation between contexts. What sort of relation differs somewhat between theories (functions, partial functions, mere relations).

- In the present paper, sentences denote partial functions from contexts to contexts. We may speak of “input” and “output” context.

---

\(^2\)This sentence sounds pretty good on a special reading, as an epistemic “inference” conditional.
• The denotation of a complex sentence is a function of the denotations of its parts (here, its clauses constituents). It is undefined for a given input context if any of the parts is undefined for its local context (see below).

• Presuppositions are conditions on the input context: If the input does not meet the conditions, the output is undefined and the interpretation cannot proceed. (Hence the denotation of the sentence is a partial function.)

More formally, in the simplest version:

i. A context is a set of worlds (i.e., a proposition in Stalnaker’s terms). Here taken to represent the common ground between the interlocutors (i.e., the worlds compatible with all the propositions that the interlocutors have agreed to treat as true for the purposes of the current conversation).

ii. Presuppositions are propositions.

iii. A context c admits a sentence S iff it entails all of S’s presuppositions.

iv. The result of updating a context c with a sentence S is written ‘c + S’. (There are many different notations in the literature.)

v. Definition of truth in terms of CCP: If context c is true in world w (i.e., w ∈ c) and c admits sentence S, then S is true in w iff c + S is true in w. (Notice that this means that a sentence whose presuppositions are false at w is not true at w. Nor, as we’ll see, is its negation.)

3.3 Recursive definitions: Propositional case.

Heim commits a certain amount of sloppiness here in using letters like A both for sentences (linguistic objects) and for the CCPs denoted by them. Moreover, she has no independent general way to refer to the set of worlds in which a sentence is true (since truth is defined in terms of CCP). For a cleaner formulation, see Section 1.1 of Muskens et al. (1997). For now, though, let’s go ahead anyway and let [A] be the set of worlds in which A is true, for any atomic sentence A. Also, \ stands for set subtraction.

\[
\begin{align*}
(21) & \quad \text{For atomic } A, c + A = c \cap [A] \\
(22) & \quad c + \neg A = c \setminus (c + A) \\
(23) & \quad c + A \land B = (c + A) + B \\
(24) & \quad c + A \rightarrow B = c \setminus ((c + A) \setminus ((c + A) + B))
\end{align*}
\]

Some discussion:

• The clause for atomic sentences should be clear enough: The new context is the set of worlds in c in which A is true.

• Negation: Remove from c all and only the worlds that “survive” an update with A.

• Conjunction: Sequential update: execute the left conjunct first on the original context, then execute the right conjunct on the resulting result.
• Conditionals: This is a bit tricky. We can visualize it as follows (‘$\overline{A}$’ stands for the negation of $A$):

<table>
<thead>
<tr>
<th>Context</th>
<th>Union of</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$(AB, \overline{A}B, AB, \overline{A}B)$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td></td>
</tr>
<tr>
<td>$c_1 = (c + A)$</td>
<td>$(A \overline{B})$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td></td>
</tr>
<tr>
<td>$c_2 = (c + A) + B$</td>
<td>$(AB)$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td></td>
</tr>
<tr>
<td>$c_3 = c_1 \setminus c_2 = (c + A) \setminus ((c + A) + B)$</td>
<td>$(A, \overline{B})$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td></td>
</tr>
<tr>
<td>$c_4 = c \setminus c_3 = c \setminus ((c + A) \setminus ((c + A) + B))$</td>
<td>$(AB, \overline{A}B, \overline{A}B)$</td>
</tr>
</tbody>
</table>

Thus we end up with the set of just those worlds in which the material conditional $A \rightarrow B$ is true.

3.4 Temporary contexts

Notice that in the course of the interpretation of some sentences (here, negation and conditionals), one or more “temporary” or contexts are created.

$\begin{align*}
    c + \neg A &= c \setminus \boxed{c + A} \\
    c + A \rightarrow B &= c \setminus \boxed{c + A \setminus (c + A + B)}
\end{align*}$

We will see later that this has two important consequences:

1. Discourse referents introduced by indefinites in these “temporary” contexts are (usually)
   i. available for anaphoric reference within the same temporal context (including more deeply embedded temporal contexts which “inherit” them.
   ii. not available for anaphoric reference in subsequent discourse.

(25)  a. John has [a car], [It] is parked outside.
      b. #John doesn’t have [a car], [It] is parked outside.\(^3\)

(26)  a. If John has [a car], [it] is parked outside.
      b. #If John doesn’t have [a car], [it] is parked outside.

Interestingly:

(27)  a. ✓ John doesn’t have [a car], [It] would be parked outside.
      b. ✗ If John didn’t have [a car], [it] {is / would be} parked outside.

There are good accounts of the goodness of (27a); I’m not aware of a good account of why (27b) is bad while (27a) is fine.

\(^3\)This sentence is fine on a “specific” reading with wide scope for the indefinite: “There is a car that John doesn’t have.” That’s not the reading that I have in mind here.
2. They persist (for a little while) and can be referred to later, serving as implicit conditional antecedents:

(28) a. A thief might come in. He would steal the silver.
b. #A thief might come in. He is armed.\(^4\)

Both (27a) and (28a) are cases of modal subordination. We’ll come back to that tomorrow.

3.5 Indefinites and quantification

Universal quantifiers are treated as one would expect. Indefinite NPs, however, are not treated as having quantificational denotations (they have “no quantificational force of their own”).

(29) a. Every nation cherishes its king.
b. A fat man was pushing his bicycle.

At the beginning of Section 3.1, Heim briefly discusses the “file card” metaphor. I must admit that I don’t find this very helpful in understanding the particular setup that she introduces in this paper. Let’s try to do without it.

- Contexts are sets of possibilities.
- Before, a possibility was a possible world – a way the world might be.
- Now, a possibility is a world combined with an assignment of values to discourse referents.
- That’s a combination of two kinds of knowledge or information: About the facts (encoded in the possible world) and about the discourse (encoded in the value assignment). (See Groenendijk et al., 1996, for more on this).
- Heim takes a variable assignment to be a function \( g : \mathbb{N} \mapsto D \), where \( D \) is the domain of individuals. Thus the number of discourse referents is countably infinite.
- Thus a context may look something like this:

\[
\begin{array}{c|ccccc}
1 & 2 & 3 & \ldots \\
\hline
a & a & a & \ldots & w \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & b & \ldots & w \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & c & \ldots & w \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & a & a & \ldots & v \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & b & \ldots & v \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & c & \ldots & v \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & a & a & \ldots & u \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & b & \ldots & u \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a & b & c & \ldots & u \\
\vdots & \vdots & \vdots & \ddots & \vdots 
\end{array}
\]

- As Heim’s definition (17) makes clear, the proposition determined by (30) is \( \{w, v, u\} \) – the set of worlds that occur in it paired with some value assignment or other.

\(^4\)Again, we are only interested in the non-specific reading. The sentence is fine on the specific reading.
Now suppose we have the following denotations:

\[
\begin{align*}
\llbracket \text{man} \rrbracket &= \begin{bmatrix} w & \mapsto & \{a, b\} \\
                          v & \mapsto & \{a\} \\
                          u & \mapsto & \{b\} \end{bmatrix} \\
\llbracket \text{happy} \rrbracket &= \begin{bmatrix} w & \mapsto & \{b, c\} \\
                           v & \mapsto & \{b\} \\
                           u & \mapsto & \{b, c\} \end{bmatrix}
\end{align*}
\]

**Indefinites**

Let’s update the context in (30) with \( \text{man}(x_2) \land \text{happy}(x_2) \). Recall that \( c + (\text{man}(x_2) \land \text{happy}(x_2)) = (c + \text{man}(x_2)) + \text{happy}(x_2) \).

Updates with formulae of this form are defined as follows (cf. Heim’s (19)):

\[
c + P(x_i) = c \cap \{\langle g, w \rangle | g(i) \in \llbracket P \rrbracket(w)\}
\]

For instance: \( c + \text{man}(x_2) = c \cap \{\langle g, w \rangle | g(2) \in \llbracket \text{man} \rrbracket(w)\} \).

\[
\begin{align*}
+\text{man}(x_2) &= \left\{ \begin{array}{c}
1 \ 2 \ 3 \ \ldots \\
a \ a \ a \ \ldots \ w \\
a \ b \ b \ \ldots \ w \\
a \ b \ c \ \ldots \ w \\
\ldots \\
a \ a \ a \ \ldots \ v \\
a \ b \ b \ \ldots \ v \\
a \ b \ c \ \ldots \ v \\
\ldots \\
a \ a \ a \ \ldots \ u \\
a \ b \ b \ \ldots \ u \\
a \ b \ c \ \ldots \ u \\
\ldots
\end{array} \right. \\
+\text{happy}(x_2) &= \left\{ \begin{array}{c}
1 \ 2 \ 3 \ \ldots \\
a \ b \ b \ \ldots \ w \\
a \ b \ c \ \ldots \ w \\
\ldots \\
a \ b \ b \ \ldots \ u \\
a \ b \ c \ \ldots \ u \\
\ldots
\end{array} \right.
\end{align*}
\]

What information is encoded in the resulting context? Two things:

1. The individual referred to by \( x_2 \) is a happy man.
   - Information about the **discourse**
     (still entailed by the context in subsequent updates).

2. We are not in world \( v \)
   - Information about the **facts**

The proposition determined by the new context after the update is \( \{w, u\} \).

- A world “survives” the update just in case there is a happy man in it.
Caveat

Despite the above desirable results, it is actually **not** a good idea to regard $\text{man}(x_2) \land \text{happy}(x_2)$ as a translation of sentences like these:

(34)  
  a. A man is happy  
  b. There is a happy man

Q: Why?  
A: Consider the negation of (34):

(35)  
  a. No man is happy.  
  b. There is no happy man.

Consider what happens when we interpret this as directed:

(36)  
$$c + (35) = c \setminus (c + \text{man}(x_s) \land \text{happy}(x_2)) =$$

The proposition determined by this is $\{w, v, u\}$. But intuitively the sentence is true only in $v$!

- The reason: $\neg(\text{man}(x_2) \land \text{happy}(x_2))$ really translates (37):

  (37)  
  $x_2$ does not refer to a happy man.

This is very different from ‘No man is happy’.

- However, this is not a problem for other dynamic systems (including Heim’s File Change Semantics; see Heim, 1983a). We’ll see later why.

For the same reason that $\neg(\text{man}(x_2) \land \text{happy}(x_2))$ is not a good translation of ‘No man is happy’, it is also not a good translation of ‘Every man is unhappy’. (Check this for yourself). So how does Heim handle universal quantification in this paper?
Universals

Indefinites are not treated as quantified sentences. However, universals are.
Now, if we were to continue updating the above context with

\[(38) \quad \text{every} \ x_2, \ \text{woman}(x_2), \ \text{tall}(x_2)\]

we would immediately get vacuous truth, since \(g(2)\) has previously been restricted to men. To avoid this, Heim ensures that a “fresh” variable is used whenever a quantifier is interpreted:

\[(39) \quad \text{For any two sequences } g, g' \text{ that differ at most in their } i\text{-th member, and for any world } w: \langle g, w \rangle \in c \iff \langle g', w \rangle \in c.\]

In words, the following must be true for all pairs in \(\langle g, w \rangle\) in \(c\): For every individual \(d\) in \(w\), the possibility \(\langle g[2/d], w \rangle\) must be present in \(c\).

Whith the same denotations as before, consider now the sentence

\[(40) \quad \begin{align*} 
\text{a. Every man is happy.} \\
\text{b. every } x_2, \ \text{man}(x_2), \ \text{happy}(x_2) \\
\text{c. } c + (\text{every } x_2, \ \text{man}(x_2), \ \text{happy}(x_2)) = \{ \langle g, w \rangle | \text{for every individual } a, \text{ if } \langle g[2/a], w \rangle \in c + \text{man}(x_2), \text{ then } \langle g[2/a], w \rangle \in (c + \text{man}(x_2)) + \text{happy}(x_2) \}\end{align*}\]

Notice that now entire “blocks” assignment-world pairs are lumped together: Either all possibilities associated with \(w\) stay, or all of them are dropped.

The contexts \(c\), \(c + \text{man}(x_2)\) and \((c + \text{man}(x_2)) + \text{happy}(x_2)\) were already derived above:

\[(41) \quad c + \text{every } x_2, \ \text{man}(x_2), \ \text{happy}(x_2) = \begin{bmatrix} 1 & 2 & 3 & \ldots \ \\
\text{a} & a & a & \ldots & u \\
\text{a} & b & b & \ldots & u \\
\text{a} & b & c & \ldots & u \\
\ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}\]

This is not to be confused with

\[(42) \quad \begin{align*} 
\text{a. If } x_2 \text{ refers to a man, that man is happy.} \\
\text{b. } \text{man}(x_2) \to \text{happy}(x_2) \end{align*}\]

which again leads to a different outcome from (41) and should therefore not be confused with ‘Every man is happy’.
In contrast to the simple system discussed here, it is usually the case in dynamic semantics that the translations for indefinites and universals are interdefinable:

\[ \neg \exists x \varphi \equiv \forall x \neg \varphi \]

As we will see, it is very simple to get this to work: Simply give the existential quantifier the role of literally introducing a new discourse referent.

4 Groenendijk and Stokhof (1991)

Dynamic Predicate Logic (DPL) is basically a variant of the approaches discussed earlier, putting the linguistic theories into a more mathematical mold.

- The language is the standard language of first-order logic.
- A model is a pair \( \langle D, F \rangle \), where \( D \) is a non-empty domain of individuals and \( F \) is an interpretation function of the non-logical constants.
- “Possibilities” are variable assignments \( g, h, \ldots \) (no world parameter involved).
- For each variable \( u \), a relation \([u]\) is defined such that for all assignments \( g, h \), \( g[u]h \) iff \( g \) and \( h \) agree on the values of all variables except \( u \) (and possibly \( u \) as well).
  \[ \text{Note: Clearly this relation is symmetric, so it doesn’t matter whether we write } g[x]h \text{ of } h[x]g. \]
  My notation will differ from the paper’s in this regard.
- An interpretation function \([\cdot]\) maps formulae to relations between variable assignments.
  \[ \text{Note: The definitions in the paper have expressions like ‘}(g, h) \in [\varphi]' \text{. I write } g[\varphi]h \text{’ instead. It’s more readable that way.} \]

Atomic formulae are mapped to a subset of the identity relation:

\[ g[R(t_1, \ldots, r_n)]h \text{ } \iff \text{ } g = h \land ([t_1]h, \ldots, [t_n]h) \in F(R) \]

The most important definition is that of the existential quantifier:

\[ g[\exists x \varphi]h \text{ } \iff \text{ } \exists k[g[x]k \land k[\varphi]h] \]
To see what the difference comes down to, let’s compare the relation from Heim (1983b) with this one. Since there is no world parameter involved, let’s just use world \( w \) from above. (This time I’ll use a simplified notation.) Consider what happens the three assignments \( \text{aaa} \), \( \text{abb} \), and \( \text{acc} \):

\[
\begin{array}{ccc}
\text{man}(x_2) & \text{happy}(x_2) \\
\text{aaa} & \text{aaa} \\
\text{aba} & \text{aba} \\
\text{aca} & \text{aca} \\
\text{aab} & \text{aab} \\
\text{abb} & \text{abb} \\
\text{acb} & \text{acb} \\
\text{aac} & \text{aac} \\
\text{abc} & \text{abc} \\
\text{acc} & \text{acc} \\
\end{array}
\]

This makes for a much nicer logic. The most relevant improvement in (47) over (46) is that now \( \text{aaa} \), which assigns \( x_2 \) to a non-man, is related by the existential quantifier to alternatives in which \( x_2 \) is assigned to a man. This is guaranteed to happen as long as there is at least one man in the model. As a result, the negation ‘There is no happy man’ or (equivalently) ‘No man is happy’ eliminates \( \text{aaa} \).

Let’s get the other definitions.

\[
\begin{align*}
g[\llbracket R_{t_1} \cdots t_n \rrbracket] h & \iff h = g \land \llbracket [t_1]_h, \ldots, [t_n]_h \rrbracket \in F(R) \\
g[\llbracket t_1 = t_2 \rrbracket] h & \iff h = g \land [t_1]_h = [t_2]_h \\
g[\llbracket \neg \varphi \rrbracket] h & \iff h = g \land \exists k [h[\varphi]k] \\
g[\llbracket \varphi \land \psi \rrbracket] h & \iff \exists k [g[\varphi]k \land k[\psi]h] \\
g[\llbracket \varphi \lor \psi \rrbracket] h & \iff h = g \land \exists k [h[\varphi]k \lor h[\psi]k] \\
g[\llbracket \varphi \rightarrow \psi \rrbracket] h & \iff h = g \land \forall k [h[\varphi]k \rightarrow \exists j [k[\varphi]j]] \\
g[\llbracket \exists x \varphi \rrbracket] h & \iff \exists k [g[x]k \land k[\varphi]h] \\
g[\llbracket \forall x \varphi \rrbracket] h & \iff h = g \land \forall k [g[x]k \rightarrow \exists j [k[\varphi]j]]
\end{align*}
\]
• As long as there is an individual with property \( P \), the update with \([\exists x P(x)]\) succeeds for all inputs \( g \), regardless of whether \( g(x) \) is such an individual or not.

• However, the output will consist only of assignments \( h \) such that \( h(x) \) has property \( P \).

**Existentially quantified formulae can map one assignment to a different one. (In the picture, they introduce diagonal arrows.) They are the only ones with that property. All others lead from an input assignment \( g \) either to \( g \) itself or nowhere at all.**

We could in fact go one step further and include expressions of the form ‘\( \exists x \)’ (for any variable \( x \)) as well-formed expressions in the language. The interpretation would be very simple:

\[
[\exists x] = d_f [x]
\]

Thus instead of \( \exists x \varphi \), we would write \( \exists x \land \varphi \).

\[
\begin{array}{ccc}
\exists x_2 & man(x_2) & happy(x_2) \\
\text{aaa} & \rightarrow & \text{aaa} \\
\rightarrow & & \\
\text{aa} & \rightarrow & \text{aa} \\
\rightarrow & & \\
\text{aca} & \rightarrow & \text{aca} \\
\rightarrow & & \\
\text{aab} & \rightarrow & \text{aab} \\
\rightarrow & & \\
\text{abb} & \rightarrow & \text{abb} \\
\rightarrow & & \\
\text{acb} & \rightarrow & \text{acb} \\
\rightarrow & & \\
\text{aac} & \rightarrow & \text{aac} \\
\rightarrow & & \\
\text{abc} & \rightarrow & \text{abc} \\
\rightarrow & & \\
\text{acc} & \rightarrow & \text{acc} \\
\end{array}
\]

Some further consequences:

1. \([\forall x [\varphi \rightarrow \psi]] = [[\exists x \varphi] \rightarrow \psi]\
2. \([\forall x \varphi] = [\exists x \rightarrow \varphi]\
3. [[\neg \exists x \varphi] = [\forall x \neg \varphi]]\

In what way does this help in accounting for anaphoric possibilities?

• Consider \( \neg \exists x [P(x)] \).

Negation is a test: The update with \([\exists x [P(x)]\]) is run on \( g \), and if it fails, then \( g[[\neg \exists x [P(x)]]]g \).

**What we get back is \( g \). And \( g(x) \) is the same individual as before – not a man.**

• Similarly for universal quantifiers and conditionals.

In the course of the evaluation, the bound variable is mapped to an individual satisfying the condition, but at the end of the day, if the sentence is true, we are left either with the input assignment \( gR \) or with nothing at all. Nothing ensures that \( g(x) \) has property \( P \), before or after the operation.

• In general: Everything except existentially quantified formulae (and conjunctions one of whose conjuncts is existentially quantified) is a test. A test has no dynamic consequences in the sense that any assertions about variables made in its course are forgotten as soon as it is over.

• Moreover: Embedding existential quantifiers into any formula except conjunction produces a test as well.
5 Heim (1983a)

File Change Semantics (FCS):
The “file card” metaphor needs a little updating. Imagine a relational database or a spreadsheet with records for each individual, recording assertions about that individual, possibly with reference to the entries of other individuals.

A search-and-update algorithm utilizes a pointer that moves around between the records, reads and writes on them.

That’s how Heim pictures what is going on in our heads during a discourse. (Pretty much inspired by Karttunen-style ideas.) See Section 1.2.1 of Muskens et al. (1997).

- A growing table like the one in (57) is a File $F$.
- A file is true iff all of its entries can be mapped to individuals in such a way that all the recorded statements are true.

This is where discourse referents, in effect, become existentially bound: As part of the interpretation procedure. In the object language (the File), their scope is “global.” Both FCS and DRT make this move. That’s how they avoid dealing with the scope of existential quantifiers.

(58) [A woman]$_1$ catches [a cat]$_2$. [It]$_2$ scratches [her]$_1$.

a. \[
\begin{array}{c}
  \text{x}_1 \text{ is a woman} \\
  \text{x}_2 \text{ is a cat}
\end{array}
\]
b. \[
\begin{array}{c}
  \text{x}_1 \text{ is a woman} \\
  \text{x}_2 \text{ is a cat}
\end{array}
\]
c. \[
\begin{array}{c}
  \text{x}_1 \text{ is a woman} \\
  \text{x}_2 \text{ is a cat}
\end{array}\text{ catches } x_2
\]

\[
\begin{array}{c}
  \text{x}_1 \text{ is a woman} \\
  \text{x}_2 \text{ is a cat}
\end{array}\text{ is caught by } x_1
\]
d. \[
\begin{array}{c}
  \text{x}_1 \text{ is a woman} \\
  \text{x}_2 \text{ is a cat}
\end{array}\text{ catches } x_2 \text{ is scratched by } x_2
\]

Formally, a file $F$ has two parts:

i. $\text{Dom}(F)$, the domain of $F$: The set of discourse referents active in $F$.

ii. $\text{Sat}(F)$, the satisfaction set of $F$: The set of assignments (functions from discourse referents to individuals) that make the file true.

iii. Sentences operate on the file, transforming it in one or both of two ways: Introducing new discourse referents into the domain and/or adding statements about discourse referents.

Sentences are decomposed into atomic assertions, possibly embedded in quantificational structures. The $i$-th such atomic assertion is of the form ‘$x_{i_1}Rx_{i_2}\ldots x_{i_k}$’. I will drop the $i$ and just write ‘$R(x_1, \ldots x_k)$’.

Indefinite and definite NPs do not denote quantifiers. They just mention discourse referents. Their use is regulated by the novelty and familiarity conditions:

(59) a. If the referent is [+def], it is already in $\text{Dom}(F)$. This is the case, for instance, with definite NPs, and pronouns.
b. If the referent is \([-def]\), it is not yet in \(Dom(F)\). This is the case, for instance, with indefinite NPs.

Linguistically, Heim treats these requirements as **presuppositions** and assumes that the update is undefined if they are not satisfied. (Which presumably is the reason for the infelicity of the corresponding sentences.)

(60) If \([R(x_1 \ldots x_k)](F)\) is defined, then

a. \(Dom([R(x_1 \ldots x_k)](F)) = Dom(F) \cup \{x_1, \ldots, x_k\}\)

b. \(Sat([R(x_1 \ldots x_k)](F)) = \{a|dom(a) = Dom(F) \cup \{x_1, \ldots, x_k\} \land \exists b \subseteq a[b \in Sat(F)] \land \langle a(x_1), \ldots, a(x_n)\rangle \in I(R)\}\)

- (60a) simply says that any referents not yet in the domain are added to the domain. That’s easy.
- (60b) is a bit more tricky. Recall that the elements of the satisfaction are functions from discourse referents to individuals. Each such element \(a\)

  i. maps all discourse referents (including the new ones) to individuals.

  This is how Heim introduces **random assignment** (not reassignment): When a new referent is introduced, all possible ways of mapping it to an individual are multiplied out.

  ii. is an extension of some member of the satisfaction set in \(F\).

  This merely preserves the information contained in \(F\).

  iii. makes the new statement true. Notice that we switched to \(I\) as the interpretation function of the model.

  This enforces the truth of the assertion about the new referent(s).

Suppose \(D = \{a, b, c, d\}\);
\(I(\text{woman}) = \{a, b\}; I(\text{cat}) = \{c, d\};\)
\(I(\text{catch}) = \{(a, c), (c, d)\}; I(\text{scratch}) = \{(c, a)\}\)

Start with \(F = \langle \emptyset, \emptyset \rangle\).

(61) \([\text{woman}(x_1)](F) = \begin{bmatrix} x_1 \\ a \\ b \end{bmatrix}\)

(62) \([\text{cat}(x_2)] \left( \begin{bmatrix} x_1 \\ a \\ b \end{bmatrix} \right) = \begin{bmatrix} x_1 & x_2 \\ a & c \\ a & d \\ b & c \\ b & d \end{bmatrix}\)

(63) \([\text{catch}(x_1, x_2)] \left( \begin{bmatrix} x_1 & x_2 \\ a & c \\ a & d \\ b & c \\ b & d \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_2 \\ a \\ c \end{bmatrix}\)
Notice that even though both the underlying metaphor and the pictures usually associated with Heim’s FCS look like records, of what is asserted about the individuals, there is no such thing in the structures we are dealing with. Only the individuals and the range of assignments consistent with the discourse information are present. That the woman caught the cat is not “written” somewhere, but can only be inferred from the fact that the two respective discourse referents are restricted to assignments for which the ‘catch’-relation holds.

In this respect, Heim’s FCS differs fundamentally from Kamp’s DRT.

6 Kamp (1981)

For a much more thorough introduction, see Kamp and Reyle (1993), as well as the many derivatives, such as Asher’s SDRT etc.

- The pictures one typically sees in DRT papers look pretty much like the ones in FCS. Discourse Representation Structures (DRSs) are built in a stepwise fashion:

$$P^n(x_1, \ldots, x_n) \in \gamma$$
$$x_1 = x_2 \in \gamma$$
$$x_1, \ldots, x_n, C_1, \ldots, C_m \in \gamma$$
$$K_1, K_2 \in \kappa$$

- More formally: We define the sets $\gamma$ of conditions and $\kappa$ of boxes simultaneously. Assume the usual sets of individual variables and $n$-ary constants.

Conditionals are translated into conditional DRSs:
(71) If a woman catches a cat, it scratches her.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
\text{woman}(x) & \text{cat}(y) \\
\text{catch}(x,y) & \Rightarrow \text{scratch}(y,x) \\
\hline
\end{array}
\]

So are universally quantified sentences:

(72) Every cat scratches a woman.

\[
\begin{array}{|c|}
\hline
y \\
\hline
\text{cat}(y) & \Rightarrow \begin{array}{|c|}
\hline
x \\
\hline
\text{woman}(x) \\
\text{scratch}(y,x) \\
\hline
\end{array} \\
\hline
\end{array}
\]

Both of (71), (72) have in common that anaphoric pronouns in subsequent discourse cannot refer back to the referents introduced locally within the embedded structures.

(73) a. If a woman catches a cat, it scratches her. #{She screams. / It is angry.}
    b. Every cat scratches a woman. #{She screams. / It is angry.}

Similarly for negation:

(74) John doesn’t own a car.

\[
\begin{array}{|c|}
\hline
x \\
\hline
x = \text{John} & \neg \begin{array}{|c|}
\hline
y \\
\hline
\text{car}(y) \\
\text{own}(x,y) \\
\hline
\end{array} \\
\hline
\end{array}
\]

(75) John doesn’t own a car. #{It is blue.}

However, if ‘a car’ has wide scope, the representation looks different:

(74’)

\[
\begin{array}{|c|}
\hline
x & y \\
\hline
x = \text{John} & \neg \begin{array}{|c|}
\hline
\text{car}(y) \\
\text{own}(x,y) \\
\hline
\end{array} \\
\hline
\end{array}
\]

The difference is determined by the translation algorithm, which maps parsed sentences to DRSs.

• Semantics: The following definitions are adapted from Muskens et al. (1997), who in turn got it from Groenendijk and Stokhof (1991). I write \( \circ \) for relation composition:

(76) \[ gR_1 \circ \ldots \circ R_nh \iff \exists k_1, \ldots, k_{n-1}[gR_1k_1 \land \ldots k_{n-1}R_nh] \]

For each referent \( x \), let the relation \([x]\) of “resetting” \( x \) be defined thus:

(77) \[ g[x]h \iff g \subseteq h \land \text{dom}(h) = \text{dom}(g) \cup \{x\} \]
The format of the following definitions makes it very clear that DRT and DPL are pretty much the same thing. (For detailed proofs, see Groenendijk and Stokhof, 1991.)

\[ g[P^n(x_1, \ldots, x_n)]h \iff h = g \land (\llbracket x_1 \rrbracket_h, \ldots, \llbracket x_n \rrbracket_h) \in I(P) \]

\[ g[x_1 = x_2]g \iff h = g \land \llbracket x_1 \rrbracket_h = \llbracket x_2 \rrbracket_h \]

\[ g[\neg K]h \iff h = g \land \neg \exists k[h[K]k] \]

\[ g[K_1 \lor K_2]h \iff h = g \land \exists k[h[K_1]k \lor h[K_2]k] \]

\[ g[K_1 \Rightarrow K_2]h \iff h = g \land \forall k[h[K_1]k \rightarrow \exists j[k[K_2]j]] \]

\[ g[\llbracket x_1 \ldots x_n | \gamma_1, \ldots, \gamma_m \rrbracket]h \iff g[x_1] \circ \ldots \circ [x_n] \circ [\gamma_1] \circ \ldots \circ [\gamma_m]h \]

It can be shown that these definitions are pretty much equivalent to the ones from Groenendijk and Stokhof (1991) above. As before: You are not supposed to use discourse referents that have not been properly introduced. Unlike DPL, however, DRS is like FCS in dealing with partial assignments of individuals to the set of “locally active” discourse referents. As a result, if a condition refers to a referent that is neither in the domain of the input assignment nor introduced in the current DRS, the result is simply undefined.

**Modality**

After Kamp’s original proposal, DRT was developed further into many directions. Among them is a modalized version in which DRSs are evaluated relative to possible-worlds models, rather than just assignments. This allows for the addition of various modal operators into the box language.

Scope relations with modals are handled a bit like the case for negation:

(84) A thief might break in. #He is tall.

(85) A thief might break in. He is tall.

This will become important tomorrow.

7 **Wrap-up**

- We have looked under the hood of the most standard dynamic frameworks.
- Ideally, you should now be able to read the primary literature yourself without getting stuck on basic formalities.
- Be sure to talk to me if you want to discuss any of this further.
- Tomorrow, we’ll look at modality.

That’s it. See you tomorrow.
References


