Lectures on Modality, Day 3

Stefan Kaufmann, Northwestern University

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Kyoto University

1 Introduction

Two topics today:

1. The application of modal logic to time
2. A detailed study of some modal and temporal expressions in English

Preliminary remarks:

§1 Modal logic is called "modal," but it is applicable in many areas besides the analysis of modality. In particular, any system that involves "change" is probably a good model for modal logic. For instance:

- Finite-state machines (Linguistics, Computer Science, Electrical Engineering)
- Stochastic processes (Speech recognition, Business, Logistics)
- Planning, Decision Theory (Pragmatics, Economics, Artificial Intelligence)
- Tense and time
- and much more ...

2 Standard tense logic

We start with a fairly simple logical apparatus which will allow us to introduce some basic notions. Nowadays, many authors, especially in the treatment of natural-language expressions, employ more complex formalisms which differ more or less dramatically from the language discussed here. In the discussion of Kaufmann (2005b), we will see one example of such approach: A first-order logic with quantification over worlds and times (such an approach is briefly discussed on page 40 of the Gamut book). However, it is still good to start with the classical system in order to get feel for the basic workings.

2.1 Syntax

§1 The language of (propositional) tense logic, like that of modal logic, includes the standard language $\mathcal{L}_\mathcal{A}$ of propositional logic. This time we extent it with four operators:

(1) If $\varphi \in \mathcal{L}_\mathcal{A}$, then $\mathbf{P}\varphi, \mathbf{H}\varphi, \mathbf{F}\varphi, \mathbf{G}\varphi \in \mathcal{L}_\mathcal{A}$

§2 The intended reading is the following (highlighting the quantificational part):

$\mathbf{P}\varphi$: φ has been true at *some* past time

$\mathbf{H}\varphi$: φ has been true at *all* past times

$\mathbf{F}\varphi$: φ will be true at *some* future time

$\mathbf{G}\varphi$: φ is going to be true at *all* future times

§3 We already see that **P** and **F** correspond to $\Diamond$, whereas **H** and **G** correspond to $\Box$. As before, we get arbitrarily complex sentences with embedded operators etc.

2.2 Semantics

§1 We will follow the standard approach in tense logic in making some basic assumptions:

- Time consists of point-like *moments* or *instances.*

  $\boxed{\Rightarrow \text{The model is built on a non-empty set of temporal instances.}}$

  Remarks: These moments are the tense-logical analog to the "possible worlds" of modality. Sentences receive truth values relative to these moments.

We don't assume that they have any duration or internal structure; we are only interested in their *location* (in time) relative to each other.

- Time has a *direction*: Given two distinct moments in time, one is *earlier* than the other

  $\boxed{\Rightarrow \text{The model has one accessibility relation: “earlier than.”}}$

- Time is not circular (you don't go more than once through the same moment).
- For now, we also assume that time is *linear* (there is no time with more than one past or future). (We will drop this assumption when we integrate with modality.)

§2 A *model* for our tense logic is a triple $\mathcal{M} = \langle T, <, V\rangle$, where

a. T is a non-empty set;
b. $< \subseteq T \times T$ is *transitive*, *irreflexive*, and *connected*; and
c. $V : T \mapsto (\mathcal{L}_{\mathcal{A}} \mapsto \{0, 1\})$ is a truth assignment (defined below).

§3 Remember the properties of relations $<$ is

- *transitive:* For all $t, t', t'' \in T$, if $t < t'$ and $t' < t''$ then $t < t''$.
- *irreflexive:* For all $t \in T$, $t \not< t$.
- *connected:* For all $t, t' \in T$, either $t < t'$ or $t' < t$.

A reasonable additional assumption that is often is that $<$ is

- *dense:* For all $t, t' \in T$, if $t < t'$ then there is some t'' such that $t < t'' < t'$.

Furthermore, it follows that $<$ is

- *asymmetric:* For all $t, t' \in T$, if $t < t'$ then $t' \not< t$.

  Proof: Suppose there are $t, t' \in T$ with $t < t'$ and $t' < t$. Then $t < t$ by transitivity, violating irreflexivity.

A connected relation is *linear*; and since ours is also irreflexive, no moment is earlier than itself. This is as it should be.

§4 We will occasionally write '$\leq$' for *'earlier than or equal to'*, and '$>$' for *'neither earlier than nor equal to'*

§5 The interpretation of the standard propositional part of $\mathcal{L}_{\mathcal{A}}$ is as usual. Here are only the clauses for the temporal operators:

(2) a. $V_t(\mathbf{P}\varphi) = \begin{cases} 1 \text{ if for some } t' \text{ s.t. } t' < t, V_{t'}(\varphi) = 1 \\ 0 \text{ otherwise} \end{cases}$

b. $V_t(\mathbf{H}\varphi) = \begin{cases} 1 \text{ if for all } t' \text{ s.t. } t' < t, V_{t'}(\varphi) = 1 \\ 0 \text{ otherwise} \end{cases}$

c. $V_t(\mathbf{F}\varphi) = \begin{cases} 1 \text{ if for some } t' \text{ s.t. } t < t', V_{t'}(\varphi) = 1 \\ 0 \text{ otherwise} \end{cases}$

d. $V_t(\mathbf{G}\varphi) = \begin{cases} 1 \text{ if for all } t' \text{ s.t. } t < t', V_{t'}(\varphi) = 1 \\ 0 \text{ otherwise} \end{cases}$

§6 As we did with modality, we can now say that the denotation of a sentence is the set of times at which it is true:

(3) $[\![\varphi]\!]^M = \{t \in T | V_t(\varphi) = 1\}$

Then the logical connectives again correspond to set-theoretic operations, as before.

2.3 A first-order alternative

As the Gamut text points out, this standard framework inherits some weaknesses from modal logic. The following includes a brief preview of readings to come.

§1 One of these problems stems from the fact that modal operators "shift" the time of evaluation: The interpretation of a sentence of the form $\mathbf{F}\varphi$ at a time t involves the interpretation of φ at a later time t'; during this step, the original time t is completely lost—in the sense that φ cannot contain any expressions whose reference depends on t. Everything is evaluated from the perspective of t'.

§2 Among other things, this creates problems with *indexicals* like '*now*', '*tomorrow*' etc.[1] Consider (4):

(4) a. I will never be as happy (again) as I am now.

b. $\neg\mathbf{F}$(I am as happy as I am now)

§3 Under the above definitions, the truth conditions for (4) are as follows:

(5) $V_t(\neg\mathbf{F}(\text{I am as happy as I am now})) = 1$ iff there is no $t' \in T$ such that $t < t'$ and $V_{t'}(\text{I am as happy as I am now}) = 1$

[1] An exact analog of this problem occurs in the modal domain with words like '*actually*'.

Although we keep track of t and its relation to t' in (5), we do so only in the metalanguage. The sentence that is evaluted at t' cannot refer to t. However, (6) sounds like a tautology; how could it be false?

(6) $V_{t'}$(I am as happy as I am now) =?

§4 The idea of adding a designated time t_0 to the model, briefly discussed by Gamut is helpful, but does not really solve the problem. The model would then no longer account for the fact that *'now'* means different things at different times: (7) is sometimes true and sometimes false.

(7) It is 5pm now.

But in a model with one designated time, its truth value would be constant.

§5 Another way of dealing with the problem is to abandon the language of modal logic altogether and use instead a first-order language with variables ranging over times.

§6 I will not give a fully explicit definition for this language. It has variables of two *sorts* (with disjoint domains: individuals and times), a symbol for the *'earlier-than'* relation (I will use '$<$') and a way of predicating sentences of times. The Gamut text (p. 40) gives the following examples:

(8)

$P(t_0)$	Mary sings
$\exists t(t_0 < t \wedge P(t))$	Mary will sing.
$\exists t(t < t_0 \wedge P(t))$	Mary sang.
...	

§7 In (8), we see that this still relies on the presence of the designated time t_0. We can do better than that by binding the evaluation time of the outermost sentence to an operator.

§8 Recall that $[\![\varphi]\!]^M$ is a set of sentence. We can alternatively spell out the denotation of a sentence as the *characteristic function* of that set, using the λ-operator; the simplest way to do this is (9) (dropping the superscript 'M'):

(9)
$$\begin{aligned} [\![\varphi]\!] &= \lambda t[V_t(\varphi)] \\ [\![\mathbf{P}\varphi]\!] &= \lambda t \exists t'[t' < t \wedge V_{t'}(\varphi)] \\ [\![\mathbf{F}\varphi]\!] &= \lambda t \exists t'[t < t' \wedge V_{t'}(\varphi)] \end{aligned}$$

§9 If s, t are the types of times and truth values, respectively, the denotation of the sentence is a function of type $\langle s, t\rangle$, the analog of a "proposition" in the temporal realm.

§10 This does not mean that we have already solved the problem of (4). But we have made it a *'linguistic'* problem. If $\varphi =$ *'I am as happy as I am now'*, we have to ensure (by the way we set up the compositional semantics) that *'now'* co-refers with the outer variable t. This can be done, although it is still not easy.

§11 More on this later. There will be more Lambdas.

3 Modality and time

Two main issues:

1. the asymmetry between past and future; and
2. the relationship between epistemic (subjective) and "metaphysical" (objective) uncertainty.

3.1 Some linguistic motivation

We shall briefly consider two pieces of English data: Conditionals and the English Present tense.

3.1.1 Counterfactual conditionals

The following is a famous example due to Adams (1970):

(10) a. If Oswald had not killed Kennedy, someone else would have.
b. If Oswald does not kill Kennedy, someone else will.
c. If Oswald did not kill Kennedy, someone else did.

§1 (10a,b) both report *predictions* from the perspective of a past time.

§2 It is not clear how such examples could be analyzed if we did not have some representation of alternative courses of events—histories which are different from the actual one but were possible (in some sense) at the time in question.

§3 The difference between (10a,b) appears to be little more than one of tense: (10a) *is false now* because (10b) *was false then* (say, on November 21, 1963).[2]

§4 Unlike (10a,b), (10c) is *true*. Intuitively, this is because we know that Kennedy is dead.

§5 All three of (10a,b,c) quantify over alternative "histories."

§6 (10a,c) are evaluated from the perspective of the speech time, based on everything that is known about the past.

§7 One thing that is known is that Kennedy was assassinated. This fact makes (10c) true, but it does not affect the falsehood of (10a).

[2]This intuition holds for these examples, but the general story is more complicated. See Kaufmann (2005a) and references therein for more discussion.

§8 Some courses of events (those in which Oswald did not kill Kennedy) were <u>*possible futures*</u> on Nov. 21, 1963 (and our judgments about (10a,b) are sensitive to them).
But they are not <u>*possible pasts*</u> now (therefore (10c) is false).

⇒ The progress of history involves the loss of open possibilities.

More on conditionals

The following is related to discussion by Edgington (1995).

(11) a. If this piece of granite is placed in water, it will dissolve.
b. If this piece of sugar is placed in water, it will dissolve.

§1 There are clear intuitions about the truth values of (11): (11a) is false and (11b) is true.

§2 Moreover, these truth values are *objective*, based on the physical properties and dispositions of the material in question.

§3 Accordingly, we would readily reject someone's assertion of (11a) as *false* (and that of (11b) as *true*).

§4 Now suppose the item in question was in fact *not* placed in water, and consider (12).

(12) a. If this piece of granite was placed in water, it dissolved.
b. If this piece of sugar was place in water, it dissolved.

§5 It is not clear that (12a,b) even *have* truth values. Nor do the physical properties of the material make any difference.

§6 This is readily explained in terms of *presupposition failure*: Natural-language quantifiers (such as conditionals) presuppose that the domain of quantification is non-empty. Since the antecedent is false, there is "no ideal, objective thing to think" about such conditionals, Edgington (1995) puts it.

§7 *However*, someone's assertion of (12a) or (12b) does not therefore become automatically false or meaningless. It is still a sensible statement about the speaker's belief state, *as long as the speaker does not know whether the material was placed in water.*

§8 On a second look, (11a,b) also have this doxastic reading, *in addition* to their "objective" truth values.
This latter reading, to highlight the contrast with the doxastic (subjective) one, is sometimes called "metaphysical" (or objective).

- Doxastic: relative to the future courses of events which the speaker considers possible.
- Metaphysical: relative to the future courses of events that are *actually* possible.
- Some assumptions that I won't argue for here:
  - The past and present are objectively "fixed." Past facts *cannot* now be different from what they actually are. (Although they *could have* been different.)
    Thus metaphysical (objective) alternatives at time t share the same past up to and including t. There is no *objective* uncertainty about the past.
  - The future is objectively "open."
    Thus metaphysical alternatives at time t may differ among each other.[3]
  - ⇒ This explains why (11a,b) have a "metaphysical" reading and (12a,b) have none: There were antecedent-worlds in the metaphysical modal base before the time in question, but there are none afterwards.
  - However, past and present are not subjectively "fixed." Not all past facts are known.
    Thus doxastic (subjective) alternatives at t may differ with regard to the past. There can be *subjective* uncertainty about the past.
  - The future is subjectively "open." Objective uncertainty implies subjective uncertainty: What is objectively not (yet) settled cannot (already) be known.
  - ⇒ Thus we have doxastic (epistemic) readings for both (11) and (12). However, only for (11) does it make sense whether the beliefs on which the assertion is based are "correct."

3.1.2 The English Present tense

(13) a. Megan was in her office yesterday.
b. Megan is in her office (now).
c. Megan is in her office tomorrow.

§1 The Present tense in English can be used with both present and future reference times. Thus as far as temporal reference is concerned, the Present complements the Past.

[3] This presupposes that the world is non-deterministic.

Based this criterion, it would appear that there are two tenses in English, Past and Non-Past, illustrated by (13a) on the one hand and (13b,c), on the other.

§2 However, while there is general agreement that (13a,b) differ only in tense and temporal reference, most authors maintain that the "futurate" Present in (13c) differs rather dramatically from both.

§3 Specifically, for the truth of (13c) it is not sufficient that a certain state obtain in the future, but this occurrence has to be "pre-determined" or "scheduled," in some sense, at speech time. No such connotation is observed in either (13a) or (13b).

§4 Notice that the sentences in (13) are stative. A slightly different pattern is observed with non-stative predicates:

(14) a. Megan came to her office yesterday.
b. Megan comes to her office (now).
c. Megan comes to her office tomorrow.

Here, both (14b) and (14c) call for the "scheduling" reading.

§5 In addition to this contrast, there is a difference in temporal reference: (13b) asserts that a state holds at speech time, whereas the event in (14b) is asserted to occur in the (near) future. This is due to the aspectual properties of the sentences.

§6 *Observation:*
The reference time *precedes* the evaluation time in (13a,b) and in (14a). These three sentences do not carry the "scheduling" reading.
The reference time *follows* the evaluation time in (13c) and in (14b,c). These three sentences do not carry the "scheduling" reading.

- The assumption that English has only two tenses (Past and Non-past) could be maintained if the addition "scheduling" connotation in (13c) and (14b,c) can be explained independently.
- Kaufmann (2005b) claims that the relation between evaluation time and reference time is the key to such an independent explanation.

4 Technicalities

§1 The preceding discussion suggests that we should build our models in such a way that at each moment in time, there may be different possible continuations into the future, but there can be only one past.

§2 The discussion of the English Present has already brought up to competing notions of "truth" with future reference in such models:[4]

Ockhamist Truth: Truth in *one* particular future continuation (namely, the actual one).

Peircean Truth: Truth in *all* future continuations. This is often called *settledness*.

§3 *Remarks:*

- Ockhamist truth accounts for uncertainty about the future while keeping the principles of Excluded Middle ($\varphi \vee \neg\varphi$ is always true) and Non-Contradiction ($\varphi \wedge \neg\varphi$ is never true).

  Under this conception, the truth value of a sentence about the future is *already* determined (and has been determined throughout the past), but cannot be *known* in advance, until enough alternatives have been ruled out.

- Peircean truth accords more with the way sentences are actually *used*.

  Here, future sentences are either false or lack a truth value (depending on the author) until the facts are in.

- A sentence is true in the Peircean sense iff it is settled in the Ockhamist sense. But Ockham-truth cannot be defined in Peircean terms (without reference to future times).

§4 We can skip the definitions of "treelike" models.

§5 We will focus on $T \times W$-models. Here are the main definitions; they will be discussed in connection with the Kaufmann (2005b) handout.

Definition 1 (T × W-frame—Thomason, 1984)
A $T \times W$-frame is a structure $\langle W, T, <, \approx \rangle$, where W and T are disjoint nonempty sets; $<$ is a transitive relation on T which is also irreflexive and linear; and $\approx$ is a relation in $T \times W \times W$ such that (i) for all $t \in T$, $\approx_t$ is an equivalence relation; (ii) for all $t, t' \in T$ and $w, w' \in W$, if $w \approx_t w'$ and $t' < t$ then $w \approx_{t'} w'$.

[4] The terms are Prior's. See also Burgess (1979).

- Recall that an equivalence relation is reflexive, symmetric and transitive. At each time t, $\approx_t$ is a *partition* of the set of world-time pairs at t. Within each cell of the partition, the world-time pairs are fully connected; however, there is no link across cells in the partition. This captures the intuition that historical alternatives at t are *indistinguishable* from each other.

- The last condition ensures that as time progresses, things do not *become* possible. This condition encodes the idea that there is no uncertainty about the past.

Definition 2
Given a $T \times W$-frame $\langle W, T, <, \approx\rangle$, let $I = W \times T$. The relations $<$ and $\approx$ are extended to $I \times I$ as follows:
a. $\langle w, t\rangle \approx \langle w', t'\rangle$ if and only if $w \approx_t w'$ and $t = t'$;
b. $\langle w, t\rangle < \langle w', t'\rangle$ if and only if $w = w'$ and $t < t'$.

Definition 3 (History model)
Let $\mathcal{A}$ be a set of propositional variables. A history model *for $\mathcal{A}$ is a structure $\mathcal{M} = \langle W, T, <, \approx, V\rangle$, where $\langle W, T, <, \approx\rangle$ is a $T \times W$-frame and $V : \mathcal{A} \mapsto (I \mapsto \{0, 1\})$ is a truth assignment for $\mathcal{A}$ such that for all $A \in \mathcal{A}$ and $i, j \in I$, if $i \approx j$ then $V(A)(i) = V(A)(j)$.*

- This is pretty much Thomason's definition, changing the notation from his '$||\varphi||^h_{w,t}$' to '$V_{\langle w,t\rangle}(\varphi)$'. (Our V corresponds to his h.)

- The last condition ensures that historical alternatives are not distinguished by the atomic sentences in the language. This had better be the case, for we want them to be indistinguish*able*.

- Jointly with the condition (ii) in Def. 1, which ensured the monotonic loss of alternatives through time, this implies that two worlds $w.w'$ such that $w \approx_t w'$ agree not on the truth values of all atomic sentences, but indeed all sentences which do not refer to the future.

Definition 4 (Historicity and lack of foreknowledge)
An accessibility relation R in $I \times I$ is
a. modal *if and only if $\langle w, t\rangle R \langle w', t'\rangle$ implies $t = t'$;*
b. temporal *if and only if $\langle w, t\rangle R \langle w', t'\rangle$ implies $w = w'$.*
A modal accessibility relation R
a. is historical *if and only if iRj and $i \approx k$ jointly imply kRj;*
b. lacks foreknowledge *if and only if iRj and $j \approx k$ jointly imply iRk.*

- If you imagine a picture in which time flows from left to right and worlds are lined up vertically, the modal relations extend vertically and temporal relations horizontally. The paper (Kaufmann, 2005b) argues that the interpretation of certain conditionals requires relations that extend “diagonally,” obtained as composites of modal and temporal relations.

- This second part allows us to place constraints on the interaction between $\approx$ and other accessibility relations. A historical relation is subject to historical necessity. A relation that lacks foreknowledge cannot “cut across” the cells induced by the partition $\approx_t$. With regard to the doxastic relation $\sim$, this means that what is not yet settled cannot already be known.

Definition 5 (Doxastic history)
Given a history model $\langle W, T, <, \approx, V\rangle$, a doxastic history *is a modal relation $\sim$ in $I \times I$ that is historical and lacks foreknowledge, and such that*
a. $\sim$ is transitive, serial and euclidean;
b. if $i \sim j$, $i' < i$ and $j' < j$, then $i' \sim j'$.

- Clause (a) is a slightly weaker condition than the corresponding one for $\approx$: Here, $\sim$ need not be reflexive, thus beliefs can be false. (However, the agent who holds those beliefs wouldn't know.)

- Clause (b) corresponds to the monotonicity requirement in Def. 1. It is less realistic here. It implies that previously held beliefs are never revised or forgotten. This is clearly an oversimplification, but it lets us avoid a lot of intricate technicalities.

Definition 6 (Modal base)
A modal accessibility relation $R \subseteq I \times I$ is a modal base *if and only if it is historical.*

Remark 1
For any accessibility relation R:
a. $\Box_{\approx}\Box_R\varphi \Longrightarrow \Box_R\varphi$;
b. $\Box_R\Box_{\approx}\varphi \Longrightarrow \Box_R\varphi$.

Remark 2
An accessibility relation R
a. is historical if and only if $\Box_R\varphi \Longrightarrow \Box_{\approx}\Box_R\varphi$;
b. lacks foreknowledge if and only if $\Box_R\varphi \Longrightarrow \Box_R\Box_{\approx}\varphi$.

Definition 7 (Presumption of decidedness)
A sentence φ *is* presumed decided *by an agent* a *at* $i \in I$ *if and only if* $\Box_{\sim^a}(\varphi \rightarrow \Box_{\approx}\varphi)$ *is true at* i*.*

- Intuitively, agent a holds this attitude knows that the truth value of φ is objectively settled, but (s)he does not know which way. Recall $\sim$ at i accesses a union of equivalence classes under $\approx$. So the agent's belif state may contain cells in which φ it is true and others in which it is false. However, within each cell, the truth value is constant.

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