

# Lectures on Modality, Day 2

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## 1 Conversational backgrounds

The earlier sections were mainly concerned with the formal logical background. Kratzer (1981) adopts many of these notions in her analysis of natural-language modal expressions. However, there is one difference in detail that we need to be aware of: The modal base represented as a function which, for each world, returns a set of *propositions*, not a set of worlds.

- <sup>s 1</sup> A *Conversational background* is a function  $f : W \mapsto \wp(\wp(W))$ , i.e., from worlds to sets of propositions (i.e., sets of sets of worlds).
- <sup>s 2</sup> There are various kinds of conversational backgrounds: totally realistic, epistemic, stereotypical, deontic, “circumstantial,” etc. (Recall that there is some variation and debate in the literature; no exhaustive list.)
- <sup>s 3</sup> Each conversational background can serve as a *modal base*: For a given world  $w$ , the set of accessible worlds is simply  $\bigcap f(w)$ , the the intersection of the propositions in  $f(w)$ .

Notice:

$$\bigcap \{\emptyset\} = \emptyset$$

$$\bigcap \emptyset = W$$

$$\bigcap \{W\} = W$$

Kratzer’s “empty” conversational background is the constant function  $f_e$  such that for all  $w \in W$ ,  $f_e(w) = \emptyset$ .

- <sup>s 4</sup> *Note*: What we earlier called “modal base” was a *set of worlds* and would correspond to  $\bigcap f(w)$ . Kratzer alternatively calls  $f$  itself the “modal base.” The term is used in both senses in the literature.

## 2 Ordering sources

So far we have discussed two parameters of variation in the semantics of modality: *modal base* and *modal force*. We now turn to the third major building block of Kratzer’s (1981) theory: *Ordering Sources*.

Ordering sources have been used in the analysis of the following phenomena:<sup>1</sup>

1. Weak readings of necessity modals
2. Graded modality
3. “Normative” preferences between circumstantial possibilities
4. Practical inference with contradictory preferences
5. Indicative conditionals
6. Counterfactual conditionals

### 2.1 Some motivating linguistic facts

Before we turn to the formal representation of ordering sources, we’ll look at two of the phenomena that prompted Kratzer to employ them in the first place.

#### 2.1.1 Weak readings of necessity modals

See also Kratzer (1981, p. 56/57) for related German examples.

- (1) a. This is the road to Springfield.  
b. This must be the road to Springfield.

<sup>s 1</sup> ‘*must*’ clearly “feels like” a necessity modal.

<sup>s 2</sup> But intuitively, (1b) is semantically *weaker* than (1a): You can believe (and assert) (1b) without believing (or asserting) (1a), but not *vice versa*.<sup>2</sup>

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<sup>1</sup>This list is not exhaustive. Kratzer (1981) discusses all of them except indicative conditionals.

<sup>2</sup>Kratzer’s argument is slightly different: She notes that under a “‘pure’ epistemic interpretation (presumably a realistic one) of the modal, (1b) entails (1a). This is true, but the premise that (1b) has such a “‘pure’ epistemic reading is not so obvious to me.

<sup>s 3</sup> Kratzer wants to treat ‘*muss*’ as a necessity modal *and* account for the fact that it gives rise to relatively weak readings. Ordering sources help her accomplish this.

<sup>s 4</sup> Some Japanese epistemic modals have a similar behavior:

- (2) a. *p*-ni chigai nai  
b. *p*-hazu da  
c. ...

### 2.1.2 Graded modality

So far, we’ve had two modal forces: Necessity (universal quantification over the modal base) and possibility (existential quantification over the modal base). Kratzer now labels them *simple* necessity and possibility, in contrast to the following:

- human necessity  
(cf. (1a), (1b) above)
- human possibility  
(‘*may*’; ‘*can*’; ...)
- slight possibility  
(‘*might*’ etc.; far-fetched but possible)
- comparative possibility (*p* is more likely than *q*)

These distinctions cannot be drawn in terms of universal and existential quantification alone. Ordering sources make it possible.

## 2.2 Technicalities

<sup>s 1</sup> Basic idea: Impose an *order* on the modal base and let the quantification range only over the minimal elements of this order.

<sup>s 2</sup> Formally: An order  $\leq_{g(w)}$  on the modal base  $\cap f(w)$  is derived from a second conversational background—the *ordering source*  $g(w)$ .

<sup>s 3</sup> In general: Let  $\Phi$  be any set of propositions. Then define the order as follows:

$$w \leq_{\Phi} z \text{ iff } \{p | p \in \Phi \wedge z \in p\} \subseteq \{p | p \in \Phi \wedge w \in p\}$$

<sup>s 4</sup> *Note:*

- $\leq_{\Phi}$  is naturally read “less than or equal,” but here  $w \leq_{\Phi} z$  means that  $w$  verifies *all* the propositions in  $\Phi$  that  $z$  verifies, and possibly more. Some people find this a bit counterintuitive. This is also the reason why I said above that the quantification ranges over the *minimal* elements, although some might prefer to call them the *maximal* elements.
- This order does not give us “degrees” of conformity with the ordering source. We only get a way of *comparing* worlds and telling which is better/worse; there is no absolute measure of “goodness” associated with the worlds.
- $\leq_{\Phi}$ , for any  $\Phi$ , is a *pre-order* (transitive and reflexive).

<sup>s 5</sup> With the order in place, we make the modals sensitive to the order. Kratzer’s definition is quite complicated:

- $p$  is a *human necessity* in a world  $w$  with respect to a modal base  $f$  and an ordering source  $g$  if, and only if, the following condition is fulfilled: For all  $u \in \bigcap f(w)$  there is a  $v \in \bigcap f(w)$  such that
    - $v \leq_{g(w)} u$  and
    - for all  $z \in \bigcap f(w)$ : If  $z \leq_{g(w)} v$ , then  $z \in p$ .
- (3) In words: For all worlds  $u$  in the modal base, there is a world  $v$  in the modal base that is at least as close to the ideal and that is not equalled or outranked by any world  $z$  in the modal base in which  $p$  is false.
- (4) In still other words: As you “hop across worlds” closer and closer to the ideal, you will eventually reach a point at which between you and the ideal there are only  $p$ -worlds.

Q: Why so complicated?

A: Because we can’t be sure that there is a “best” world. Indeed, we can’t even be sure that there is a “best” *set* of worlds: If the modal base and ordering source are both infinite (and nothing prevents this), there may be an infinite sequence of worlds getting closer and closer to the ideal without ever reaching it. This is best illustrated with counterfactual conditionals, so we’ll return to it below. Meanwhile, see Lewis (1973, 1981) for detailed discussions of these issues.

<sup>s 6</sup> Once we have “human” necessity and possibility, we can do away with “simple” necessity and possibility altogether: They come out as special

cases with the empty ordering source, in which case the order imposed on the modal base is fully connected (i.e., connecting all worlds in the modal base with all others).

<sup>s 7</sup> Such pre-orders over sets of possible worlds are also quite popular in Artificial Intelligence (less so their derivation from ordering sources). See in particular the discussion on “relative likelihood” in Halpern (2003) and its relationship to other representations of uncertainty, such as probability and possibility.

Back to the phenomena Kratzer is trying to explain. . .

### 2.3 Weak necessity modals

This is quite clear now: Epistemic ‘*must*’ denotes “human” necessity, not “simple” necessity. It signals that the speaker’s judgment is (at least partly) based on assumptions other than established facts. Some of the worlds that are, strictly speaking, compatible with what the speaker knows, may lie outside the domain of (modal) quantification.

Kratzer correctly points out that this can turn a realistic (reflexive) modal base into a non-realistic basis for the evaluation of the modal: The actual world may be compatible with the speaker’s beliefs, without being among those closest to the ideal. (pp. 56/57)

### 2.4 Graded modality

Kratzer, pp. 48–50:

‘ <i>Es kann gut sein, daß. . .</i> ’	Human possibility
‘ <i>There is a good possibility that . . .</i> ’	
‘ <i>Es besteht eine geringe Möglichkeit, daß. . .</i> ’	Slight possibility
‘ <i>There is a slight possibility that. . .</i> ’	
‘ <i>Es kann eher sein, daß. . . als daß. . .</i> ’	Comparative possibility
‘ <i>It is more likely that . . . than that . . .</i> ’	
‘ <i>Es ist wahrscheinlich, daß. . .</i> ’	Human necessity
‘ <i>It is probable that . . .</i> ’	

<sup>s 1</sup> Human necessity: See above.

<sup>s 2</sup> Human possibility:  $\neg p$  is not a human necessity.

<sup>s 3</sup> Slight possibility:  $\neg p$  is a human necessity, but  $p$  is compatible with the modal base

<sup>s 4</sup> Comparative possibility: Kratzer’s paraphrase:  $p$  is more possible than  $q$  iff the following both hold:

- a. for every accessible  $q$ -world there is an accessible  $p$ -world which is at least as close to the ideal; and
- b. there is an accessible  $p$ -world for which there is no accessible  $q$ -world that is at least as close to the ideal.

<sup>s 5</sup> Note once again that we do not get quantitative notions here; there is no degree of goodness or closeness to the ideal.

## 2.5 Normative rankings

Above we saw that the worlds in epistemic or circumstantial modal bases can be ranked according to relative *likelihood*. But rankings based on other ordering sources (deontic, buletic, teleological etc.) are also possible. The meaning of ' $v \leq_{g(w)} z$ ' is then '*v is more desirable / more in accordance with the laws etc. at w than z*'. This is the obvious analog of likelihood in these domains.

The technical notions of Human Necessity etc. carry over to this case. However, as Kratzer shows for German, there can be subtle differences between individual verbs with regard to the ordering sources they allow. Similar facts hold for English. Here is a variation on an example from Kratzer (1991b):

- (5) a. *[Given your state of health] you should go to Davos rather than to Amsterdam.*
- b. *[Given that you like the sea more than the mountains] you should go to Amsterdam rather than go to Davos.*

Davos is an old mountain resort for patients with tuberculosis. (5a,b) may both be true (without the bracketed material), and both may be good advice, depending on which of the addressee’s preferences take precedence in the situation.

## 2.6 Practical inferences

Kratzer’s example (p. 65–67) involves a person who wants to become mayor of his town and does not want to go to the local pub. The “circumstances” (i.e., the relevant facts that constitute the circumstantial modal base) include the fact that it is impossible to become mayor without going to the pub.

<sup>s 1</sup> Let  $m =$  ‘*I become mayor*’,  $p =$  ‘*I go to the pub regularly*’.

- <sup>s 2</sup> Modal base:  $\bigcap f(w)$   
 In all worlds in  $\bigcap f(w)$ , either I don't become mayor or I go to the pub regularly.  
 This is the material interpretation of the conditional '*I become mayor only if I go to the pub regularly*'. Thus:  $\bigcap f(w) \subseteq \llbracket m \rightarrow p \rrbracket$
- <sup>s 3</sup> Ordering source:  $g(w) = \{m, \neg p\}$
- <sup>s 4</sup> Obviously there is no world in the modal base that satisfies *all* my desires:  $\bigcap f(w) \cap \bigcap g(w) = \emptyset$
- <sup>s 5</sup> However, there *are* worlds in the modal base that satisfy one of the propositions in  $g(w)$ , and they are better than those that satisfy neither:
- A: the set of worlds in the modal base in which I become mayor:  
 $\bigcap f(w) \cap m = \bigcap (f(w) \cup \{m\})$
- B: the set of worlds in the modal base in which I don't go to the pub:  
 $\bigcap f(w) \cap \neg p = \bigcap (f(w) \cup \{\neg p\})$
- C: the remainder of the modal base, where I do go to the pub and don't become mayor:  
 $\bigcap f(w) \setminus (A \cup B)$
- <sup>s 6</sup> Both  $m$  (I become mayor) and  $\neg p$  (I don't go to the pub) are “human possibilities” in this situation. (This is simply because their negations are not human necessities—that's what Kratzer shows at the bottom of page 66.)
- <sup>s 7</sup> *Note:* This analysis works because it draws a distinction between the “circumstances” (the facts that are fixed and that the subject's decisions cannot alter) and the subject's goals.  
 Deontic modals are often interpreted in this way: They provide a preference ranking over the worlds in a circumstantial or epistemic modal base.

## 2.7 Conditionals

- <sup>s 1</sup> In classical Fregean logic, '*if A, B*' is interpreted as the *material conditional* (also called material implication) ' $\rightarrow$ ':
- (6)  $A \rightarrow B$  is true iff either  $A$  is false, or  $B$  is true, or both.
- <sup>s 2</sup> The material conditional is a truth function on a par with conjunction and disjunction. However, while there is general agreement that the latter are well-suited to capture the truth conditions of '*and*' and '*or*',

the logical properties of the material conditional do not well match those of conditional sentences. For example,  $A \rightarrow B$  and  $A \rightarrow \neg B$  are mutually consistent, and the falsehood of  $A$  is sufficient for the truth of both, hence of their conjunction. But (7b) is intuitively contradictory and does not follow from (7a). Likewise, the negation of  $A \rightarrow B$  is equivalent to  $A \wedge \neg B$ , but (7c,d) are not intuitively equivalent.

- (7) a. Today is Saturday.  
 b. If today is Friday, it is raining, and if today is Friday, it is not raining.  
 c. It is not the case that if the team wins, I will be happy.  
 d. The team will win and I will be unhappy.

<sup>s 3</sup> Kratzer’s account is based on the alternative assumption, due to Lewis (1975), that *if*-clauses restrict quantifiers. In particular, at least in the examples she discusses, they restrict *modal* operators. The simplest way of implementing this would be like this:

- (8) ‘If  $A$  then  $B$ ’ is true at a possible world  $w$  relative to an accessibility relation  $R$  iff for all possible worlds  $w'$  such that  $wRw'$  and  $A$  is true at  $w'$ ,  $B$  is true at  $w'$ .

<sup>s 4</sup> Formally, conditionals (indicative and counterfactual) are interpreted with respect to modal bases and ordering sources.

<sup>s 5</sup> The role of the antecedent as a restrictor of the modal base is accomplished by adding the antecedent to the modal base  $f$ , giving a new conversational background  $f^+$ , such that: For all  $w \in W$ ,  $f^+(w) = f(w) \cup \{A\}$ , where ‘ $A$ ’ is the interpretation of the antecedent.

<sup>s 6</sup> For clarity, I prefer to write ‘ $f^{+A}(w)$ ’ to make clear (in a context-independent manner) which proposition is added. Thus the definition (still without the ordering source) becomes (9):

- (9) ‘If  $A$  then  $B$ ’ is true at a possible world  $w$  relative to a modal base  $f$  iff for all possible worlds  $w' \in \bigcap f^{+A}(w)$ ,  $B$  is true at  $w'$ .

<sup>s 7</sup> Where no overt modal is present, the modal force is necessity. By default, the modal base is epistemic.

<sup>s 8</sup> It is easy to show that the material conditional and strict implication ( $\Box(A \rightarrow C)$ , with the universal accessibility relation) fall out as special cases. See pp. 68/69.

<sup>s 9</sup> *Note:* This treatment is in line with two venerable proposals in the



philosophical literature: the “antecedent-as-restrictor” analysis of Lewis (1975), and the “Ramsey Test,” Ramsey’s (1929) quote which relates the interpretation of (indicative) conditionals to the dynamics of belief change:

(RT) If two people are arguing ‘If  $p$  will  $q$ ?’ and are both in doubt as to  $p$ , they are adding  $p$  hypothetically to their stock of knowledge and arguing on that basis about  $q$  . . . We can say they are fixing their degrees of belief in  $q$  given  $p$ .

However, Kratzer does not give us a way to implement Ramsey’s notion of “degrees of belief.”<sup>3</sup>

### 2.7.1 Indicative conditionals

So why do we need ordering sources in the interpretation of indicative conditionals?

<sup>s 1</sup> Definition (9) accounts nicely for the context-dependence of conditionals. A given conditional can be simultaneously true with respect to one modal base and false with respect to another. Thus (10) may be objectively (circumstantially) true, but believed to be false by a speaker with insufficient information or false beliefs.

(10) If this material is heated to 500°C, it will burn.

<sup>s 2</sup> However, (9), like the material conditional and the strict conditional, fails to account for the invalidity of certain *non-monotonic* inference patterns involving conditionals.

<sup>s 3</sup> For instance, under all three analyses, a true conditional remains true under Strengthening of the Antecedent (*‘If A then B’* entails *‘if C and A then B’*). But (10) can be true while (11) is false.

(11) If this material is placed in a vacuum chamber and heated to 500°C, it will burn.

<sup>s 4</sup> Kratzer’s solution: Like all modals, ‘will’ in (9) and (11) involves “human necessity” (not strict necessity), given by an ordering source (in this case, a “stereotypical” one).

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<sup>3</sup>For that, you have to turn to probabilistic treatments, which we won’t in these lectures. See Eells and Skyrms (1994); Edgington (1995); Bennett (2003) for overviews and Kaufmann (2004, 2005a,c); Kaufmann et al. (2004) for one particular proposal.

- (12) ‘If  $A$  then  $B$ ’ is true at  $w$  relative to a modal base  $f$  and ordering source  $g$  iff for every world  $w' \in \bigcap f^{+A}(w)$ , there is a world  $w'' \in \bigcap f^{+A}(w)$  such that  $w'' \leq_{g(w)} w'$  and for all  $w''' \in \bigcap f^{+A}(w)$  such that  $w''' \leq_{g(w)} w''$ ,  $B$  is true at  $w'''$ .

In other words: ‘If  $A$  then  $B$ ’ is true at  $w$  iff for every  $A$ -world  $w'$  in the modal base, there is an  $AB$ -world  $w''$  in the modal base that is at least as likely as  $w'$  and not equalled or outranked in normalcy by any  $A$ -world in the modal base at which  $B$  is false.

- <sup>s 5</sup> This offers a solution to the above problem. Suppose the material is normally not placed in a vacuum chamber. Then every antecedent-world at which it is, is outranked in normalcy by one at which it is not, thus (10) may be true while (11) is false. Technically, the conditional (13) is a human necessity.

- (13) If this material is heated to 500°C, it won’t be in a vacuum chamber.

- <sup>s 6</sup> So in effect, *even though the modal base and the ordering source are the same in both (9) and (12)*, the truth of the conditional depends on different sets of worlds.

(9): Worlds at which the material is heated and not in vacuum.

(12): Worlds at which the material is heated and in a vacuum.

- <sup>s 7</sup> *Addendum*: In a later paper (Kratzer, 1991b, p. 648), Kratzer makes more explicit that she seems to think of conditional antecedents as *modifiers* of the modal in the consequent.

- (14)  $\llbracket \text{if } \alpha, \text{ must } \beta \rrbracket^{f,g} = \llbracket \text{must } \beta \rrbracket^{f^{+\alpha},g}$ ,  
 where for all  $w \in W$ ,  $f^{+\alpha}(w) = f(w) \cup \{\llbracket \alpha \rrbracket^{f,g}\}$ .

However, she never provided a fully explicit *compositional* derivation for this interpretation. We will see one proposal in Kaufmann (2005b).

## 2.7.2 Counterfactual conditionals

- <sup>s 1</sup> Unlike indicative conditionals, counterfactuals are typically used when the antecedent is in doubt or known to be false. (There are a few exceptions, but we can ignore them.)

In such a case, the antecedent cannot be added to the modal base consistently.

- <sup>s 2</sup> Kratzer: “A counterfactual is characterized by an empty modal base  $f$  and a totally realistic ordering source  $g$ .”
- <sup>s 3</sup> Recall that  $\bigcap \emptyset = W$ . Thus the worlds in the modal base are *all* the worlds in  $W$ .
- <sup>s 4</sup> A totally realistic ordering source means that  $\bigcap g(w) = \{w\}$ . This ensures that  $w$  is a minimal element in the order induced by  $g(w)$  (i.e.,  $w$  is closest to itself): For each  $w' \neq w$ , there must be some proposition  $p$  in  $g(w)$  such that  $w \in p$  and  $w' \notin p$ . But since  $g$  is realistic, there is no  $q \in g(w)$  such that  $w' \in q$  and  $w \notin q$ .
- <sup>s 5</sup> The invalid inferences discussed above for indicative conditionals (Strengthening of the Antecedent etc.) are invalid for counterfactuals as well. Consider the famous example due to Lewis (1973):
- (15) a. If kangaroos had no tails, they would topple over.  
 b. If kangaroos had no tails and walked on crutches, they would topple over.

The fact that (15a) does not entail (15b) is taken care of in the same way as (10) and (11) above.

- <sup>s 6</sup> *Note:* This treatment of counterfactuals is closely related to the “ordering semantics” of Lewis (1973). For a comparison, see Lewis (1981). These issues are beyond the scope of this tutorial, but they are fascinating and worth studying.

This concludes the introduction to Kratzer’s semantics. In the next two sections, we will take a brief look at two topics that have risen to some prominence in later work: Deontic conditionals and the structural difference (if any) behind the root/epistemic distinction. Both of these areas are full of unresolved issues, and we won’t discuss them in great detail. The main message is that there are reasons to think that all conditionals, and perhaps all modalized sentences, involve an (overt or covert) epistemic modal with wide scope. If they contain an explicit deontic modal, that modal takes scope within the epistemic one. This is a (tentative) claim that we will see again tomorrow.

### 3 Deontic conditionals

We now turn to a potential problem with Kratzer’s account. The question it raises is: *Does Kratzer’s account work for all conditionals (as she intends)?* The answer is “no.”

<sup>s 1</sup> Recall that Kratzer’s account of conditionals crucially rests on the assumption that the modal base and ordering source for the whole conditional is the same as the one for the consequent.<sup>4</sup>

Thus for instance, if the consequent has a deontic modal, then the whole conditional is interpreted with respect to the same deontic modal base (modulo the addition of the antecedent).

**The main problem.** Frank (1996) and Zvolenszky (2002) point out an obvious problem with this account:

**Fact 1**

*If A entails B, under Kratzer’s analysis, a conditional ‘If A, must B’ is necessarily true.*

**Fact 2**

*Likewise for the conditional ‘If A, may B’, as long as there are A-worlds in the modal base.*

<sup>s 1</sup> It is not hard to see why this is so: Whatever the modal base is (epistemic, circumstantial, deontic, etc.), after adding the antecedent to it, we end up with a set of worlds ( $\bigcap f^{+A}(w)$ ) in which the antecedent is true. But if the antecedent entails the consequent, then the consequent is also true throughout  $\bigcap f^{+A}(w)$ . Furthermore, even if there is a non-trivial ordering source, the truth of the conditional still depends only on worlds in the modal base.

<sup>s 2</sup> But clearly, deontic conditionals of this form can be false. The best counterexamples are ones with root modals in which *A* and *B* are the same. Here are some examples from Zvolenszky:

- (16) a. If teenagers drink, then teenagers must drink.  
b. If teenagers drink, then teenagers may drink.

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<sup>4</sup>There is in fact a subtle difference: Both Kratzer (1981) and Kratzer (1991b) claim that the consequent is evaluated with respect to modal base  $f^{+A}$ , where *A* is the antecedent, and ordering source *g*. However, whereas Kratzer (1981) says that *f* is the modal base of the *antecedent* and somehow inherited by the consequent, in Kratzer (1991b) it is the modal base of *the whole conditional*.

- c. If I file my taxes, then I want to file my taxes.
- d. If children don't eat spinach, then children shouldn't eat spinach.

<sup>s 3</sup> All of these sentences are predicted true under Kratzer's account with respect to *any* modal base  $f$  and ordering source  $g$ .

<sup>s 4</sup> Another one of Zvolenzsky's examples: In fact, Britney Spears does (or did, at the time) have a contract with Pepsi that stipulated that (17a) is true: Among all the cola-drinking worlds, the Pepsi worlds are closer to the ideal than others.

- (17) a. If Britney drinks cola in public, she must drink Pepsi.
- b. If Britney drinks Coke in public, she must drink Coke.

<sup>s 5</sup> Under Kratzer's analysis, (17a) may well be true, but (17b) is necessarily *also* true. In fact, though, they are contradictory.

**Further problems.** We may assume that the indicative in (18a) is interpreted relative to a circumstantial modal base and a deontic ordering source.

- (18) a. If Max buys this car, he must pay taxes for it.
- b. If Max had bought a car, he would have to pay taxes for it.
- c. If Max really loved his dog, he should take it for a walk.

<sup>s 1</sup> Frank (1996) points out that under Kratzer's analysis, a similar interpretation is not available for counterfactuals like (18b,c). Kratzer states counterfactuals generally are interpreted with respect to *an empty modal base and a circumstantial ordering source*. Thus  $\bigcap f(w) = W$  and  $\bigcap g(w) = \{w\}$ .

<sup>s 2</sup> But then we are in danger of losing the deontic flavor of the conditional: The worlds in the modal base are ranked with respect to their similarity to the *actual* world, which is not (necessarily) the deontic ideal.

<sup>s 3</sup> Frank briefly considers the possibility of operating with *two* ordering sources (one realistic, one deontic) on the same modal base, but cannot think of a principled way (and neither can I) of specifying how both would interact in order to generate the order.

<sup>s 4</sup> So should we just say that counterfactuals can be interpreted with respect to a *deontic* ordering source (combined perhaps with the empty background  $f$ )?

*No.* The deontic modal base would be one based on the laws in *this* world (i.e., the world of evaluation). But what if the antecedent itself

implies that the laws are different from the actual ones?

- (19) If Luther hadn't brought about the Reformation, we would still have to pay indulgence.

**The solution.** It seems better to choose the antecedent-worlds  $w'$  first (for instance, *via* an epistemic modal base) and then evaluate the consequent with respect to the laws at  $w'$  (not  $w$ ).

<sup>s 1</sup> That is Frank's proposal. "Deontic" conditionals are really epistemic ones: The antecedent restricts an epistemic modal base; the deontic modal in the consequent does not take scope over the whole conditional.

<sup>s 2</sup> For additional evidence, Frank reminds us that an epistemic modal *can* appear in these sentences (20c,d), and that it is generally assumed that such an epistemic modal is present (overtly or covertly) in all conditionals (cf. Kratzer, 1991b,a).

Thus, Frank asks, why not just say that there is an implicit epistemic modal in (20a,b)?

- (20) If Max stays with Grandma, ...
- a. he is allowed to take the dog for a walk.
  - b. he must take the dog for a walk.
  - c. he might be allowed to take the dog for a walk.
  - d. he might have to take the dog for a walk.

<sup>s 3</sup> Frank's proposal for (20a–d) still runs into some problems. The one for (21) is better (though still not perfect):<sup>5</sup>

- (21) a. If Mad bought a car, he would have to pay taxes.  
b.  $necessary_{f+p(w)}^{g(w)}(necessary_{f'(w')}^{g'(w')}q)$   
c. "For all worlds  $w'$  in the (epistemic) modal base in which Max buys a car, the following is true: He pays taxes in all worlds  $w'' \in f'(w')$  that are closest to the deontic ideal in  $w'$ ."

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<sup>5</sup>There is some residual discussion over the nature of  $f'$ . For our purposes, we can assume that it is circumstantial, allowing for some "magic" in ensuring that it is indeed restricted to all the *relevant* worlds. Frank ultimately favors a somewhat different solution, but the details are very intricate.

To summarize, I quote: “In sum, these observations lead us to the conclusion that there are in fact no truly *deontically modalized if*-conditionals. Instead we assume conditionals with a deontic modal operator in the consequent clause to be analyzed in terms of an implicit or explicit epistemically (or circumstantially) based modal operator. The deontic modal adverb is then to be analyzed within the scope of the ‘higher’ epistemic modal operator.”

## 4 Raising vs. Control, Epistemic vs. Root

The accounts we have discussed so far do not draw any *grammatical* distinction between different kinds of modals. However, there are some arguments that such a distinction is real and semantically relevant. But these arguments are somewhat inclusive and hotly debated. In this section we will glance over some of the arguments for and against a structural difference. This survey will be open-ended, however.

<sup>s 1</sup> In particular, we are interested in the *root/epistemic* distinction. It is often assumed (see Brennan, 1993 for references) that the syntactic subjects of sentences containing modals of these classes differ in their status:

- Root modals are *control* predicates.

(22) a. Bill should leave now [*root*]  
b. [Bill should [PRO leave now]]

(23) a. Bill should be here now [*epistemic*]  
b. [*e* should [Bill be here now]]

<sup>s 2</sup> There is a clear intuition behind this:

- Root modals assert of the subject (Bill) that he has a “modal” property (here, an obligation).

This is captured in the assumption that they assign a Theta-role to the subject. The subject (and Theta-recipient) of the embedded predicate is coreferential with ‘*Bill*’ and realized as an empty placeholder PRO.

- Epistemic modals are proposition-embedding predicates. The subject stands in no special relationship with the modal. One might say that modal somehow has a “logical” subject, but this is not realized in the sentence: It is the agent who makes the judgments, most likely the speaker.

This is captured by assuming that the grammatical subject is empty. Since sentences (in English) must have subjects, the embedded predicate moves into this position on the surface. But it does not receive a Theta-role from the modal verb.

<sup>s 3</sup> *Digression on adverbs.* There is an analogous split between “speaker-oriented” and “subject-oriented” adverbs:



- *Speaker-oriented* adverbs report a judgment by the speaker (not the subject of the sentence): ‘*probably, evidently, happily, ...*’
- *Subject-oriented* adverbs, on the other hand, add a claim about the subject: ‘*carefully, cleverly, stealthily, ...*’

The speaker/subject-oriented distinction corresponds with constraints on the order of co-occurrence: Subject-oriented adverbs cannot precede speaker-oriented one (Brennan, p. 21)

- (24) Speaker-oriented > Subject-oriented: Ok.  
Probably, Max was carefully climbing the tree.
- (25) Subject-oriented > Speaker-oriented: Bad.  
??Carefully, Max was probably climbing the tree.
- (26) Speaker-oriented > Speaker-oriented: Ok.  
Happily, Max has evidently climbed the tree.

Similarly, subject-oriented adverbs cannot precede epistemic modals. However, subject-oriented adverbs can. In this regard, epistemic modals are “speaker-oriented.”

- (27) Speaker-oriented > Epistemic: Ok.  
Epistemic > Speaker-oriented: OK.  
a. Max will evidently climb the tree.  
b. Max evidently will climb the tree.
- (28) Subject-oriented > Epistemic: Bad.  
Epistemic > Subject-oriented: Ok.  
a. Max will eagerly climb the tree.  
b. ??Eagerly, Max will climb the tree.

*End of digression.*

<sup>s 4</sup> Lesson from the digression: There are good arguments that epistemic modals are *structurally* speaker-oriented (not just semantically), favoring a Raising analysis for them.

<sup>s 5</sup> However, it is much harder to argue conclusively that root modals are always subject-oriented. So it is not so clear that they are Control verbs. The arguments are a bit intricate and problematic. We’ll skip them; see Brennan for discussion.

<sup>s 6</sup> In the face of inconclusive grammatical evidence, Brennan reverts to the original intuition: That a Theta-role is assigned to the subjects of root modals (but not to those of epistemic modals).

<sup>s 7</sup> But here again, the case is clearest for epistemic modals: It is generally the speaker, not the subject, who makes the likelihood judgment or draws the inference.

- (29) a. John must be in his office right now.  
b. John may have been here this morning.

It is also reasonably clear that dynamic modals do assign Theta roles:

- (30) a. Peter can dance. (ability)  
b. Peter will sign anything he's presented with. (disposition)

However, deontic ones are a mixed bag: In (31a), the subject clearly is the bearer of the obligation, but not so in (31b).

- (31) a. You must register or else you'll get kicked out.  
b. Thesis paper must be acid-free.

<sup>s 8</sup> To account for the borderline behavior of deontic modals, Brennan appeals to the so-called *ought-to-be* vs. *ought-to-do* distinction.

**Ought-to-be:** *'It ought to be the case that...'*

The subject is not the bearer of the obligation.

- (32) There should/must/ought to be water in the tank at all times.

**Ought-to-do:** *'(S)he ought to...'*

A modal property (an obligation) is predicated of the subject.

- (33) Mary should/must/ought to put water in the tank regularly.

Notice that the expletive subject in (32) is a good sign that we do have a Raising structure here.

<sup>s 9</sup> So now the claim is that:

- On the ought-to-be reading, deontic modals are Raising, like epistemic modals;
- On the ought-to-do reading, deontics are Control (like dynamics?)

Does this salvage the account?

## Bhatt and Wurmbrand

Bhatt (1998) and Wurmbrand (1999) argue independently that despite some evidence to the contrary, even root modals are Raising verbs. Their arguments are very similar. We will go through some (not all) of Wurmbrand's.

1. Deontic modals can appear with expletive subjects.

- (34) a. There may be singing but no dancing on the premises.  
b. There must be a solution to this problem on my desk,  
tomorrow morning!

*Remark:* We've already seen this in (32) above, where it was used to argue that Ought-to-be deontics are Raising. Wurmbrand's examples are all ought-to-be, so they do not establish that *all* deontics are Raising.

Besides, Wurmbrand admits (Fn. 2) that the argument does not apply to dynamic modals. They don't occur with expletive subjects.

2. Case: In Icelandic, most subjects have Nominative Case, but some (depending on the verb) have Accusative or Dative. This phenomenon is known as "quirky case.")

- (35) Harald / \*Haraldur vantar peninga  
*Harald-ACC Harald-NOM lacks money*  
Harald lacks money.

When such subjects appear in a Control structure, the subject gets the Case of the embedding verb (usually Nominative):

- (36) Haraldur / \*Harald vonast til að vanta ekki peninga  
*Harald-NOM Harald-ACC hopes for to lack not money*  
Harald hopes not to lack money.

When they appear in a Raising structure, they get the quirky case assigned by the embedded verb:

- (37) Harald virðist vanta ekki peninga  
*Harald-ACC seems lack not money*  
Harald seems not to lack money.

With modals, these subjects behave like in Raising constructions (i.e., they keep their quirky case):

- (38) Umsækjandann verður vanta peninga  
*The-applicant-DAT must lack money*  
 The applicant must lack money (in order to apply for this grant).

Wurmbrand concludes that these modals are Raising.

*Remark:* Her example (7) is indeed deontic, but I'm not sure if it's ought-to-do, not ought-to-be.

3. Scope. In Raising constructions, a quantified subject can take scope under the embedding verb. In Control constructions, they can't. That's why we get a *de dicto* reading in (39a) (in addition to the *de re* reading), but not in (39b):

- (39) a. A student seems to walk down the hall.      [*de dicto/de re*]  
       b. A student wants to walk down the hall.      [*de re only*]

Not surprisingly, epistemic modals do that too:

- (40) A student is likely to win the lottery.      [*de dicto/de re*]

Somewhat more surprisingly, deontic modals can do it too:

- (41) a. Two Austrian skiers must win the next race (in order for either of them to win the World Cup)  
       b. An Austrian skier must win the next race (in order for Austria to have the most gold medals)

4. Theta roles. Epistemic modals do not assign theta roles. Wurmbrand argues that neither do deontic modals.

In (42a,b), the bearer of the obligation is not the syntactic subject of the sentence. But presumably something like "obligation" is the role that these verbs *would* assign if they were to assign Theta roles.

- (42) a. The traitor must die.  
       b. The old man must fall down the stairs and it must look like an accident.

Bhatt argues in a similar vein: "The distinction between an ought-to-be and an ought-to-do modality depends upon whether the obligation

is taken to be borne by someone or not.” Here the operative word is *taken to be*—Bhatt, like Wurmbrand, believes that the contrast, though intuitively real, is a pragmatic effect and that there is no evidence for a *structural* difference.

⇒ In sum, the claim that the epistemic/root distinction is syntactically Raising vs. Control looks promising at first sight, but there is not much hard evidence for it.

What remains is the intuition that epistemic modals are somehow “higher” in the tree.

This is also supported by observations on the possible orders of multiple modals the same sentence, in particular the apparently very widely applicable generalization that epistemic modals must precede all others (Cinque, 1999)—in other words, that no other modal can precede an epistemic one:

(43) Sue must have to work a lot at night.

As I said at the beginning, the discussion in this section was open-ended. My goal in bringing up this topic in the first place was, aside from interest, to prepare the ground for the claim that deontic and dynamic modals, in conditional consequents as well as otherwise, are always embedded in the scope of an epistemic modal. The question of the corresponding structural difference remains open, and I frankly admit that I don’t know the correct syntactic story either.

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